CALCULATION OF TRANSIENT FORCES DURING EMERGENCY ESCAPE OF GASES FROM AN AUTOCLAVE, WITH SPECIAL REFERENCE TO DESIGN METHODS

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SYNOPSIS

On certain chemical plants the autoclaves are protected by discharge pipes which are sealed at some distance from the outlet ends by thin metal discs. If the chemical reaction in an autoclave becomes out of control the pressure rises to a predetermined value and a disc ruptures; a rapid discharge from the autoclave then takes place.

This paper deals with various aspects of the discharge of a gas from an autoclave through a pipe. It gives an assessment of the forces generated during an unsteady discharge which must be allowed for in the design of such equipment.

The mechanism of discharge may be divided into two parts: the first is the transient-flow period during which shock waves and other finite waves are generated in the pipework and the second is the quasi-steady flow period. In the latter the flow pattern remains almost unchanged with time but the general level of pressures and discharge rates decreases progressively.

The paper deals with the calculation of forces generated during various stages of the discharge. These forces are calculated from momentum considerations and as the flow is non-steady a complication arises in the momentum equation; this is discussed and a full explanation is given.

Finally, in order to produce a safe design sample calculations based on assumed conditions for a mixture of gases are included to illustrate the techniques discussed in the paper.

Introduction

In chemical plants such as those used in the production of tetra-ethyl-lead the autoclaves are fitted with safety discs. If a chemical reaction becomes out of control the pressure in the autoclave system increases and at a predetermined value the safety disc ruptures and the vessel is rapidly ventilated. This technique protects the autoclave system from major disaster.

During the ventilating period the gas flow in the pipework is non-steady and initially strong wave-action occurs. The objects of this paper are first to describe the discharge process and secondly to outline methods for calculating the transient loads generated during the discharge. The latter are of paramount importance to the preparation of a safe design for such a plant.

The discharge process is described in some detail. First, the wave-action period which is similar to the process occurring in a simple shock tube, on which a number of text books have been published, *e.g.* Glass and Hall,[1] Gaydon and Hurle,[2] and Bradley.[3] The second is the quasi-steady flow period, which is the basis of the boundary conditions used in the theory of unsteady compressible flow. This is now well established and is treated in text books such as that of Shapiro (Vol. II)[4] and Rudinger.[5]

In gas flows of the present kind the main forces are transmitted to the walls of the duct by static pressures: this is because the shear stresses at the walls are small and in this treatment they are neglected. The forces on the ducts are given by the integrated value of the static pressure over the surfaces. As the autoclave systems are complex geometrically, in general it is not feasible to calculate the details of the flow particularly in the vicinity of the walls, in order to determine the static pressure there. A more practical alternative was therefore adopted which consisted of predicting the main features of the flow using one-dimensional unsteady gas dynamics and then calculating the forces from this description of the process. The forces were calculated using the momentum theorem and as this type of application is somewhat unusual it is discussed in detail.

Practical methods for calculating the wave action and resultant forces are presented and detailed calculations for two autoclave systems given.

The conclusions drawn from the work are that a method for calculating the transient forces in an autoclave system has been produced by extending the one-dimensional discharge calculation. It is shown that the forces generated may be very large and can change rapidly. It is therefore important that they must be taken into consideration when a plant is designed or modified.

The Process of Discharge from an Autoclave

When the safety disc bursts on a chemical plant the discharge process which follows may conveniently be described in two parts. During the first period there is strong wave action occurring in the pipes but after a succession of wave reflections and interactions the amplitudes of the waves become small. Hence the wave action period blends into the second part which is termed the quasi-steady flow period. It is convenient to discuss the two periods separately.

The wave action period

During the wave action period the flow is time-dependent and the full differential equations governing the flow are

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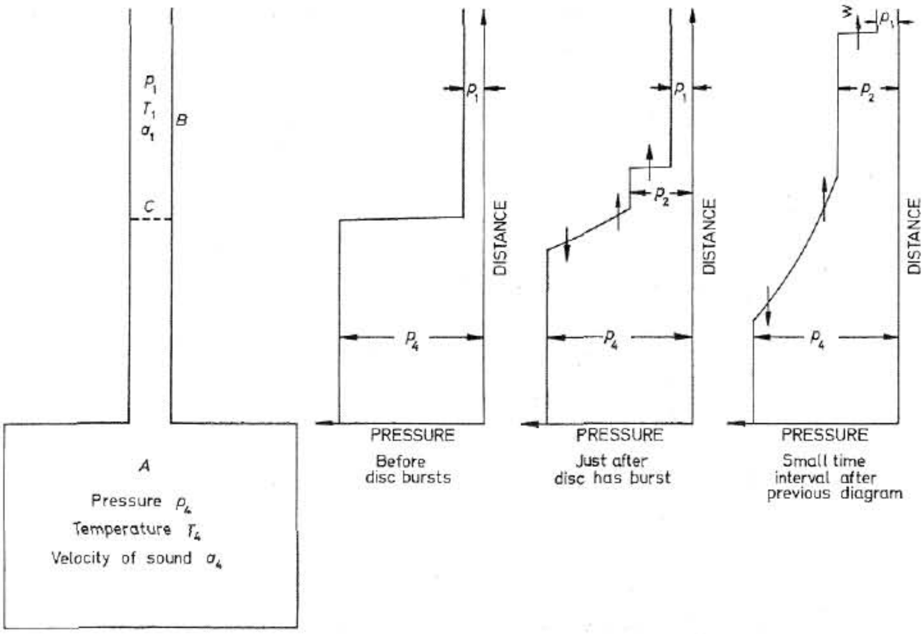

Fig. 1.—*Diagrammatic arrangement of the autoclave*

complex. However it is possible to make further simplifying assumptions without loss of a great deal of accuracy. In this connection the treatment presented considers the flow to be one-dimensional, adiabatic, unsteady, frictionless flow of perfect gases in ducts of constant cross-sectional area.

A simplified arrangement of an autoclave system is shown in Fig. 1. A represents the autoclave, B the discharge pipe, and C the safety disc. When the disc bursts a shock wave is formed which rapidly travels to the top of the discharge pipe whilst an expansion wave travels downwards towards the autoclave. The expansion wave is reflected at the end of the pipe and the reflected waves traverse the pipe and their amplitude is progressively reduced as a result of reflection and interaction.

Just after the disc bursts the flow pattern is very similar to that at the start of discharge in a shock-tube apparatus. The pressure ratio and propagation speed of the shock wave which travels towards the open end of the pipe may be calculated from the following expressions:[6]

$$\frac{p_4}{p_1} = \frac{p_2}{p_1}\left\{1 - \frac{(k_4-1)(a_1/a_4)(p_2/p_1-1)}{\sqrt{(2k_1)}\sqrt{[2k_1+(k_1+1)(p_2/p_1-1)]}}\right\}^{-2k_4/(k_4-1)} \quad (1)$$

where p is the static pressure, a the speed of sound, and $k = C_p/C_v$, ratio of the specific heat of the gas at constant pressure to the specific heat at constant volume. The subscripts refer to the regions indicated in Fig. 1.

$$\frac{p_4}{p_1} = \left(\frac{k_1-1}{k_1+1}\right)\left[\left(\frac{2k_1}{k_1-1}\right)M_s^2-1\right] \times\left[1-\frac{(k_4-1)(M_s^2-1)}{(k_1+1)(a_4/a_1)\,M_s}\right]^{-2k_4/(k_4-1)} \quad . \quad (2)$$

Here $M_s = W/a_1$ where W is the speed of the shock wave.

The speed of the gas behind the shock wave u_2 is given by the expression:

$$u_2 = a_1(p_2/p_1-1)\Big/\sqrt{\left[\frac{2/k_1}{(k_1+1)(p_2/p_1)+(k_1-1)}\right]} \quad . \quad (3)$$

The locus of the shock wave and the contact surface are plotted on the position diagram shown in Fig. 2. The contact surface is shown by a dotted line in the diagram and it represents the boundary which separates the gas originally in the pipe downstream of the safety disc and the gas which was formerly on the high pressure side of the disc.

The centred expansion wave, part of which moves into the high pressure gas and part of which is swept downstream in a supersonic flow is constructed according to the method of characteristics.[4-6] In this method waves are represented by the following two simple equations. The first is called the

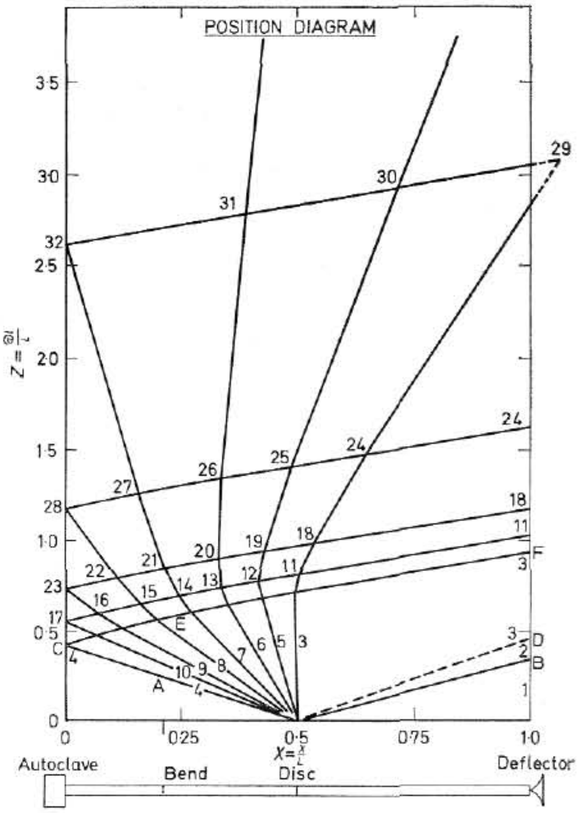

Fig. 2.—*Position diagram*

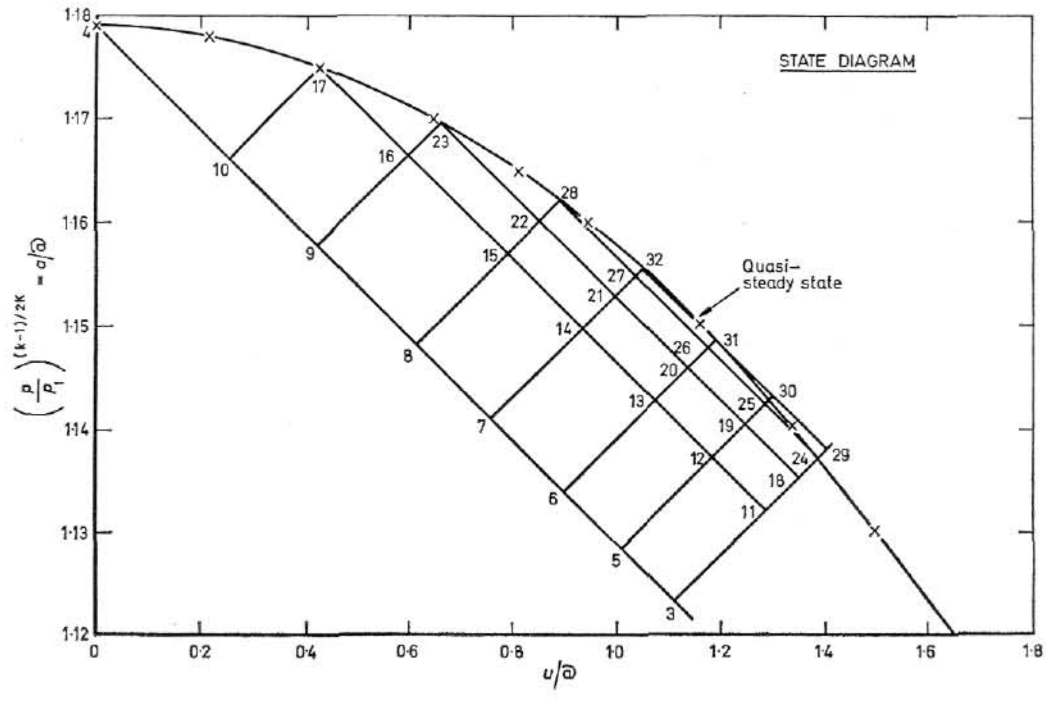

Fig. 3.—*State diagram*

direction condition and it gives the velocity of a point within the wave as:

$$\frac{dx}{dt} = u \pm a \qquad (4)$$

where x is the distance co-ordinate and t is the time. The positive sign applies to a wave moving in the positive direction of the flow whereas the minus sign applies to a wave moving in the opposite direction.

The second equation relates the state conditions within the waves and is called the compatability condition:

$$\frac{da}{du} = \mp \frac{k-1}{2} \qquad (5)$$

The method consists of solving equations (4) and (5) as finite difference equations and in doing this it should be pointed out that the plus sign in equation (4) is used with the minus sign in equation (5).

At the inlet end of the pipe the flow from the autoclave to the pipe is termed quasi-steady; that is, at a given instant of time the flow appears the same as though it were steady and over a period of time the flow pattern remains similar but the fluid properties and the mass flow-rate change progressively. The quasi-steady flow assumption at the autoclave boundary enables us to use over a short interval of time the steady flow-energy equation[7] to relate the state of the gas in the autoclave to the velocity and state just at the inlet end of the pipe. This leads to:

$$a_4^2 = a^2 + \left(\frac{k-1}{2}\right) u^2 \qquad (6)$$

Thus the a and u without a suffix in equation (6) apply to a range of conditions at the inlet end of the pipe over a short interval of time just after the rupture of the disc.

It is assumed also that the flow from the autoclave to the pipe is isentropic. This assumption neglects the loss of total pressure which occurs at entry to the pipe but simplifies the characteristic calculation for the wave action and is justified because this will tend to over-estimate wave action pressures and lead to a safe estimate of the forces involved. Work is at present in hand to develop more accurate estimates; this is discussed later in the report.

The flow from the pipe at the outlet end is in many cases supersonic during the initial wave action period and the boundary conditions do not affect the calculation. In some cases a deflector is fitted at the outlet end of the pipe; this must be designed so that it does not cause reflected waves to pass down the pipe. This aspect of the problem has been the subject of a separate investigation and is reported elsewhere.[8]

The calculation is illustrated by means of the two characteristic diagrams. Fig. 2 is the position diagram which shows the progress of the waves with time and Fig. 3 is the state diagram which gives the properties and velocity of the gas at points, lines, or regions shown by numbers on the position diagram. In these diagrams non-dimensional parameters are used as follows:

For the position diagram

$$X = \frac{x}{L} \quad \text{(distance)}$$

$$Z = \frac{@t}{L} \quad \text{(time)}$$

And for the state diagram:

$\frac{a}{@}$ non-dimensional speed of sound

$\frac{u}{@}$ non-dimensional speed of the gas.

In the above parameters L is reference length and @ is a reference speed of sound which in the present work is, for convenience, selected as the speed of sound in the gas from the autoclave when it is at a state corresponding to a reference pressure p_1 and entropy equal to that just before disc rupture, S_4. Following this definition it may be shown that:

$$\frac{a}{@} = \left(\frac{p}{p_1}\right)^{(k-1)/2k} \qquad (7)$$

It follows that the ordinate of the state diagram represents a monotonic function of pressure.

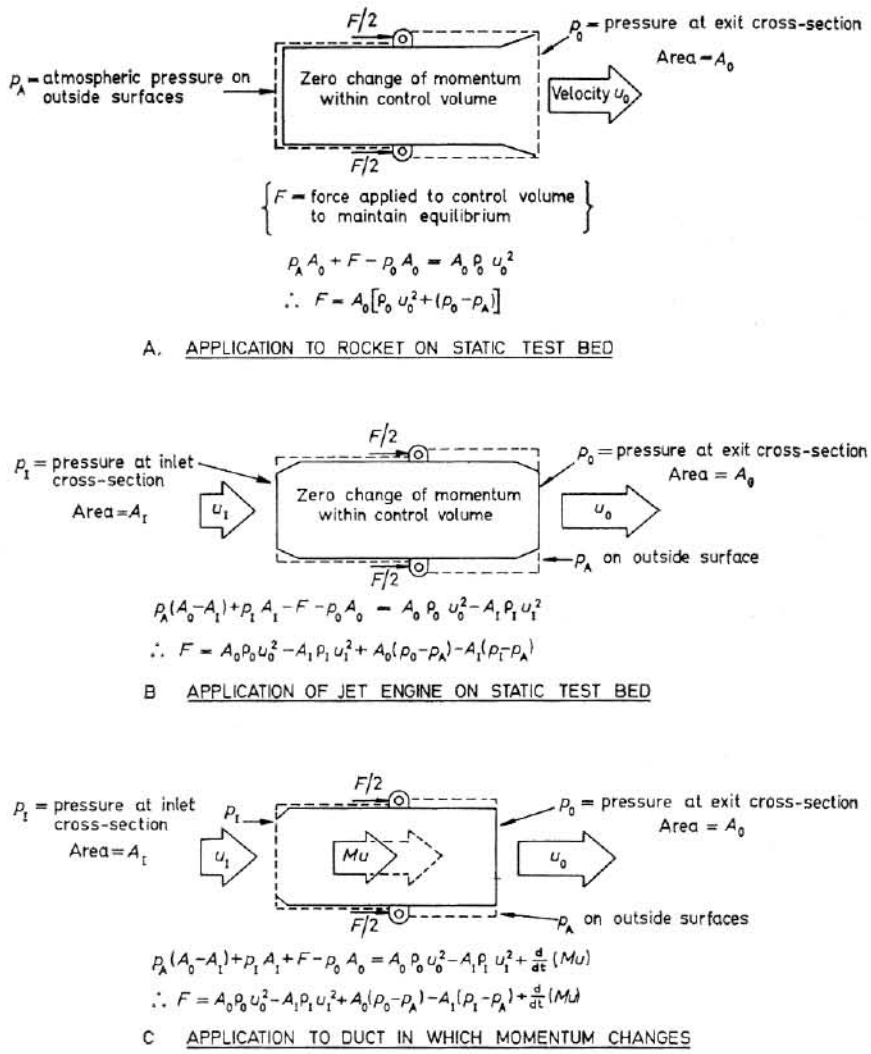

Fig. 4.—*Applications of the momentum equation. (Control surfaces shown by dashed lines*

The calculation is carried out by constructing the two diagrams of Figs 2 and 3 in a step-by-step manner. Further details of this calculation are given in Appendix I.

The quasi-steady flow period

As the wave action period continues, the mesh points in the state diagram converge towards the point of intersection of the steady flow energy ellipse given by equation (6) and the line $a = u$. When this point is reached it corresponds to quasi-steady flow and the flow in the pipe is sonic at an equal pressure all along its length. This description is very approximate because of the assumptions made to perform the wave action calculation. As the quasi-steady flow calculation is much simpler than that for the wave action it is feasible to formulate a more realistic description of the flow. It is possible on this type of calculation to take account of the viscous shear stresses at the walls of the pipe and also make some allowance for the loss of total pressure at the entry to the pipe and, in the case of a deflector causing some blockage at the pipe outlet, allowance for this can be made.

The quasi-steady type of calculation is important in estimating the rate at which the gases are discharged from the autoclave so that this is taken in conjunction with the conditions controlling the chemical reaction to ensure the venting rate is adequate. Although this apsect is really beyond the scope of the present paper it is mentioned because the discharge rate calculated during the wave action period will tend to give an over estimate of the actual value. Further details of the methods which can be used for the quasi-steady flow period are discussed by Shapiro (Vol. I).[4]

Momentum Considerations

One of the main objects of this paper is to discuss the nature of the forces generated during the discharge process from an autoclave system. As one of the main methods for calculating these forces depends upon momentum considerations, it is appropriate to discuss the momentum equation in some detail.

General case

It is shown by Shapiro (Vol. I)[4] that the momentum equation for a control volume is:

$$\Sigma \mathbf{F} = \frac{\partial}{\partial t}\left(\int_{cv} \rho \mathbf{U}\, \mathrm{d}V\right) + \oint_{cs} \rho(\mathbf{U}.\mathrm{d}\mathbf{A})\,\mathbf{U} \quad . \quad (8)$$

where $\Sigma \mathbf{F}$ is the sum of all the forces exerted by the surroundings on the material instantaneously occupying the control volume. In the present work body forces such as gravity are unimportant and only the surface forces are included in the summation. The density is denoted by ρ, the volume by V. The velocity of the material is a vector $\mathbf{U}$ and an element of area at the surface of the control volume d**A** is also a vector quantity. In *words* the equation (8) in the present case may be simply stated as:

$$\begin{pmatrix}\text{The sum of the forces}\\ \text{acting on the surface}\\ \text{of the control volume}\end{pmatrix} = \begin{pmatrix}\text{Rate of increase of}\\ \text{momentum within the}\\ \text{control volume}\end{pmatrix} + \begin{pmatrix}\text{Excess rate of}\\ \text{momentum flux leaving}\\ \text{the control volume}\end{pmatrix}.$$

In other words the forces acting on the control volume are used to provide an increase in momentum within the control volume and also increase the momentum of the stream flowing through it.

The form given in equation (8) is the most general and is complicated because it is three dimensional and also non-steady. In a large number of problems in chemical and mechanical engineering the flow is steady and in this case the first term on the right-hand side of equation (8) becomes zero; the momentum equation is well known in this form. We may however achieve some simplification without restricting our attention to steady flow.

Cases when the flow in and out of the control volume is in one direction

UNSTEADY FLOW IN A STRAIGHT PIPE

If we restrict our attention to forces and fluid flows in one direction only the momentum equation is considerably simplified and three cases are shown in Fig. 4. The first two are well known but the complication which arises in case C is due to the momentum within the control volume changing with respect to time.

In all the cases illustrated in Fig. 4 it is assumed that the flow at cross-sections I and O takes place in the horizontal direction and the trunnion thrust, F, maintains the rocket, jet engine, or pipe at rest on its test stand or static mounting. In the cases of the rocket and the jet engine it is assumed that the flow velocities inside the control volume remain constant with time. It may be assumed that the propellant and oxidant in case A and the fuel in case B enters the control volume in a vertical direction.

In case C an important part of the trunnion force, F, may occur as a result of the momentum changes occurring within the control volume. If the duct accelerates the effect of this must be included in the term $\frac{\mathrm{d}}{\mathrm{d}t}(Mu)$.

During the wave action period of discharge from an autoclave system the momentum content of a given length of pipe changes rapidly. Hence in order to calculate the forces transmitted by a length of pipe the rate of change of momentum within the pipe must be accounted for. However, during the quasi-steady flow period, the wave action has disappeared and although the momentum content of a given pipe or component may change, its rate of change is small and may be neglected.

Applications of the Momentum Theorem to Estimate Forces on Components

As explained previously the application of the momentum equation may be divided into two parts. In the first, the rate of change of momentum within the control volume is important whereas in the second, it is not. This division is followed here. It is convenient to place the control volume inside the pipes and containers, and we now consider forces exerted by the gas on the pipes and containers. These forces are equal in magnitude but opposite in direction to those exerted on the gas.

Case 1: When the rate of change of momentum within the control volume is not small

UNSTEADY FLOW IN A STRAIGHT PIPE

If a straight pipe is considered and the viscous shear stresses at the pipe walls are neglected, the only way gas forces are transmitted to the pipe is by means of the static pressures. As the static pressures are balanced no axial forces are transmitted to the pipe due to the motion—steady or unsteady—of the gas. It should be mentioned however that the pressure forces at the ends of the pipe produce the increase of momentum flux passing through the pipe plus the rate of increase of momentum within the pipe according to the momentum equation.

SIMPLE AUTOCLAVE SYSTEM JUST AFTER DISC RUPTURE

The problem considered here is to determine the forces transmitted to the surroundings external to the control volume for the brief interval of time between the disc bursting, which is assumed to be instantaneous, and the arrival at an end of the pipe of the shock wave or the expansion wave.

A control volume is selected inside the autoclave and the discharge pipe as shown in Fig. 5A. When the disc is removed instantaneously the static load that was on the disc and

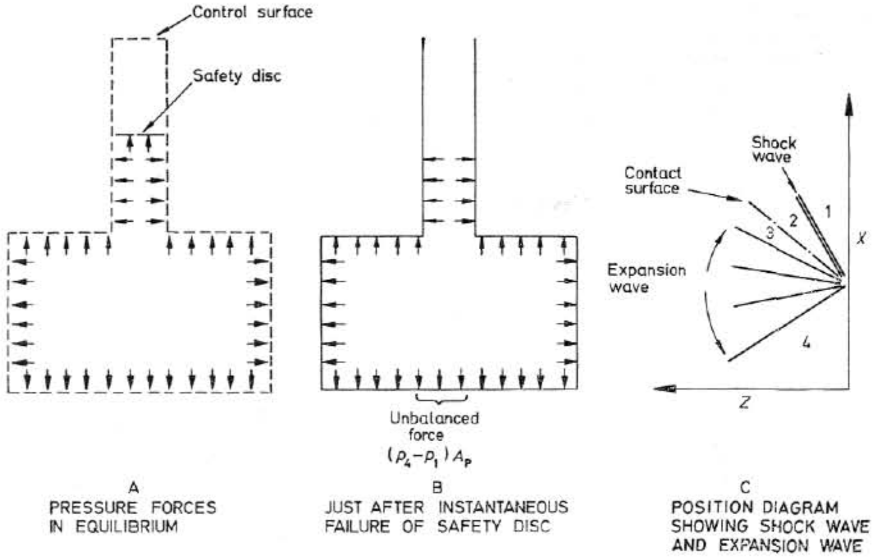

Fig. 5.—*Forces on an autoclave*

transmitted to the wall of the pipe is suddenly removed from the latter. Stress waves are set up in the pipe, a tensile stress wave travels towards the open end of the pipe, and a compressive type of stress wave travels towards the autoclave. The latter wave tends to relieve the tensile stress already existing in the pipe from its static loading. A very complex stress wave pattern develops and exists for a very short period of time. The speed of propagation of the stress waves in the metal is at least an order of magnitude higher than the fluid waves inside the pipes. Because of this it is proposed to assume the pipes and autoclave are rigid and that the load due to the pressure defect shown in Fig. 5B. appears at the base of the autoclave immediately the disc is ruptured. The magnitude of the force due to the pressure defect is $(p_4-p_1)\,A_p$. In non-dimensional terms for the example illustrated in Fig. 2 this is:

$$(p_4-p_1)\,A_p/A_p\,p_1 = p_4/p_1-1 = 44{\cdot}3-1 = 43{\cdot}3$$

Alternatively, we may calculate this force using the momentum equation. Clearly in practice the force is calculated from the pressure defect method. The following result is given only to demonstrate that the application of the momentum equation leads to the same result.

During the time interval between disc rupture and the arrival of the first wave at one end of the pipe, there is no flow in or out of the control volume. The momentum equation therefore simplifies to:

Force
= Rate of change of momentum within the control volume.

The right-hand side is evaluated in three parts, the gas between the shock wave and the contact surface (condition 2); next the gas between the contact surface and the head of the expansion wave (condition 3); and finally the gas within the expansion wave (conditions 3 to 4).

Over the time period concerned, the momentum increases as a linear function of time. The total momentum was calculated at time $Z = \frac{1}{6}$ and as the momentum is zero at $Z = 0$, the required rate of change of momentum is therefore equal to the calculated total divided by the time interval.

This calculation gave the following result:

$$\left(\frac{\text{Rate of change of momentum}}{p_1\,A_p}\right) = 43{\cdot}2.$$

This compares very favourably with the value quoted above, which clearly demonstrates the equivalence of the two methods.

SHOCK WAVE MOVING AROUND A BEND IN A PIPE

Flow in a bend with wave action has been examined experimentally by Benson, Woollatt, and Woods[9] and it was concluded that a circular bend has negligible effect on the wave action but that a sharp right angle bend does cause a pressure loss. In the following paragraphs it is assumed that we may neglect the effect of the circular bend on the wave action including the shock wave.

With reference to Figs 6A and 6B, by applying the momentum equation it may be shown that when the shock wave is approaching the bend the gas forces exerted on the pipe are:

$$\left.\begin{aligned} Fy &= (p_F-p_1)(1-\cos\theta)\,A_p \\ Fx &= -\,(p_F-p_1)(\sin\theta)\,A_p \end{aligned}\right\} \quad . \quad . \quad (9)$$

When the shock wave has moved around the bend, the forces change to:

$$\left.\begin{aligned} Fy &= (p_B+\rho_B\,u_B{}^2-p_1)(1-\cos\theta)\,A_p \\ Fx &= -\,(p_B+\rho_B\,u_B{}^2-p_1)(\sin\theta)\,A_p \end{aligned}\right\} \quad . \quad (10)$$

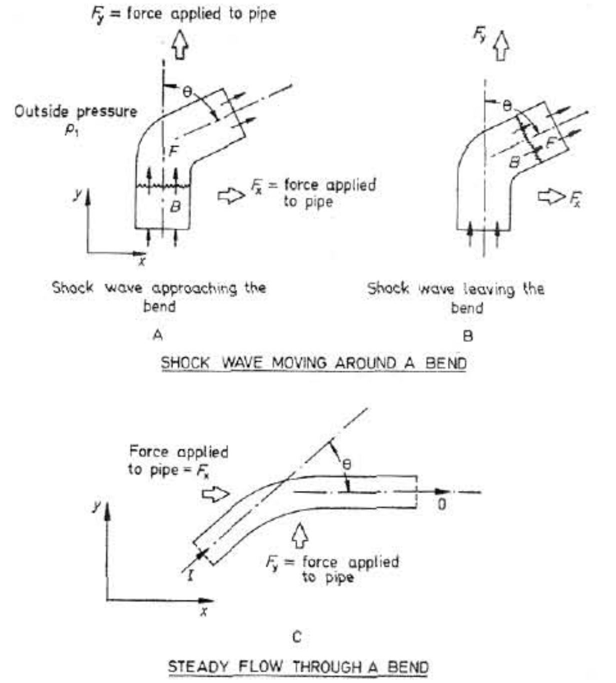

Fig. 6.—*Forces exerted on a pipe by gas*

Hence, when there is wave action in a bend of a pipe, although the outlet and inlet conditions may not change over the interval of time concerned, the loading on the pipe can change by a significant amount. A similar result is obtained when a contact surface moves through a bend.

The calculation of the total momentum in a pipe may be very involved. However, the following section includes a technique which enables us to avoid computing rates of change of momentum, as described in this section, by selecting control volumes around bends as small as possible.

Case 2: When the rate of change of momentum within the control volume is not important

It is clear that for a straight pipe, if the viscous shear stresses at the pipe walls are neglected, a flow, steady or unsteady, cannot induce any axial load on the pipe. The important pipe components to consider are therefore the two ends and any bends, the intervening straight sections contributing nothing to the force.

If the pipe shown in Fig. 6C is considered it may be shown that the application of the momentum equation leads to the following results:

$$\left.\begin{aligned} \frac{Fx}{A_p\,p_1} &= \left(\frac{f_I}{A_p\,p_1}-1\right)\cos\theta - \left(\frac{f_0}{A_p\,p_1}-1\right) \\ \frac{Fy}{A_p\,p_1} &= \left(\frac{f_I}{A_p\,p_1}-1\right)\sin\theta \end{aligned}\right\} \quad (11)$$

where the impulse function $f = (\rho u^2+p)\,A_p$, suffix $_0$ denotes outlet and suffix $_I$, inlet. If the inlet and outlet velocity and state conditions are the same, these equations reduce to:

$$\left.\begin{aligned} \frac{Fx}{A_p\,p_1} &= -\left(\frac{f}{A_p\,p_1}-1\right)(1-\cos\theta) \\ \frac{Fy}{A_p\,p_1} &= \left(\frac{f}{A_p\,p_1}-1\right)\sin\theta \end{aligned}\right\} \quad . \quad . \quad (12)$$

Hence when there is a steady or quasi-steady flow through a bend the external forces can be calculated from the geometry

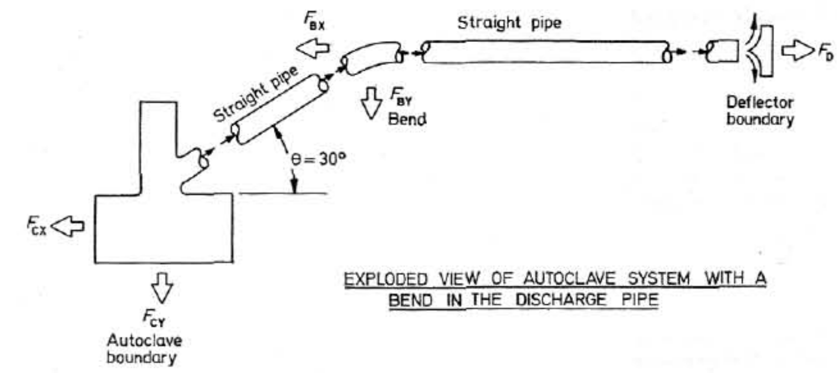

Fig. 7.—*Exploded view of an autoclave system with a bend in the discharge pipe*

of the bend and the impulse functions at the inlet and outlet.

When the flow is not quasi-steady, such as the shock wave travelling around the bend considered in the previous section, it *may* still be permissible to neglect the effect of wave action in the bend. The requirement is that the wave action period is very short, *i.e.*, the time taken from the shock wave entering to leaving the bend must be short. In this case the loading on the bend is considered to change instantaneoulsy from that given by equation (12) for the flow pattern before the shock wave arrived to that given by the same equation for the flow pattern after the shock wave has moved away from the bend. This technique also applies to a contact surface moving through a bend, and is illustrated in the examples on the complete autoclave systems discussed in the next section.

The function $f/A_p p_1$ may be expressed in terms of the parameters used in the state diagram of the method of characteristics as follows:

$$\frac{f}{A_p p_1} = \left(\frac{a}{@}\right)^{2k/(k-1)} + k\left(\frac{a}{@}\right)^{2/(k-1)} \left(\frac{u}{@}\right)^2. \quad (13)$$

Having discussed the calculation of forces in component parts of an autoclave system we next propose to combine them in order to determine the transient loading on a complete system.

Transient Forces on a Complete Autoclave System

In this section an outline is given of the gas loadings imposed on an autoclave system which includes a bend and a deflector plate.

The configuration considered is shown in Fig. 7 as an exploded view. It is visualised as being in five parts,

(1). Autoclave boundary.
(2). Straight pipe.
(3). Bend.
(4). Straight pipe.
(5). Deflector boundary.

The calculations to determine the wave action and the estimation of the forces in the component parts which are given in Appendix I have already been discussed; hence it is proposed only to treat the summation of the forces on the components.

The calculations presented deal only with the horizontal forces and it should be appreciated that in a full design calculation both the vertical forces and the bending moments must also be taken into account when stress calculations are performed.

The horizontal gas force, F_{CX}, is exerted on the autoclave boundary; F_{BX} is that on the bend, and F_D that on the deflector. These forces have been evaluated in non-dimensional terms as given by equation (13). The results of the wave action calculation and force estimations are given in Tables I and II and the corresponding force time graphs shown in Fig. 8. The two characteristic diagrams shown in Figs 2 and 3 apply to the case discussed in this section.

Various events are shown on these diagrams; first, at time zero, a load is suddenly imposed on the autoclave and bend as the disc bursts; next at A the head of the expansion wave meets the bend and the force on the bend is then gradually reduced; at B the shock wave reaches the deflector and the force on the deflector suddenly increases to a constant value while the gas at state 2 flows through the deflector. At C the expansion wave reaches the autoclave boundary and the force on the autoclave begins to change gradually. At D the contact surface reaches the deflector and the force on the deflector suddenly reduces on account of sudden change of fluid passing through the deflector. This is because the fluid from the autoclave region 3 has a lower density than that in region 2.

At point E the head of the reflection of the expansion wave reaches the bend and this causes the force which has to be exerted on the bend to increase gradually.

At point F the first part of the reflection of the expansion wave reaches the deflector and this then causes a gradual change to occur in the deflector force.

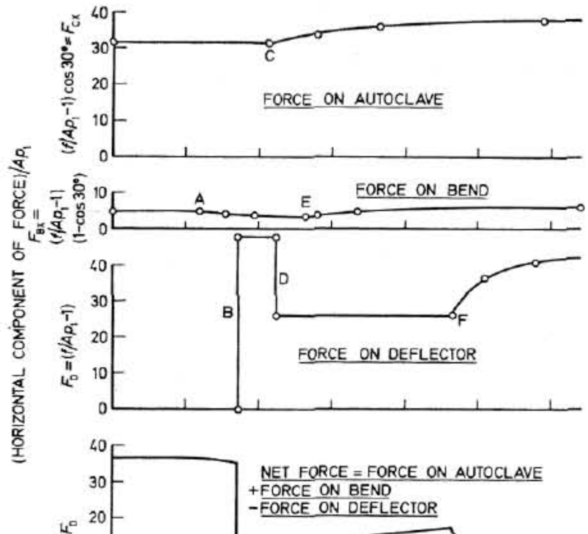

Fig. 8.—*Forces on autoclave/pipe-work system*

The net force on the complete system is shown in Fig. 8 and it will be observed that this force approaches zero as the quasi-steady state is reached.

A final check on the calculation for the complete system is that the area under the graph of horizontal force against time should equal the magnitude of the total horizontal momentum set up in the pipework when the quasi-steady state is reached. This result follows from the integration of the momentum equation for the whole system with respect to time, up to the commencement of the quasi-steady state.

Concluding Remarks

The main conclusions drawn from the paper are that a method for calculating the transient forces on an autoclave system during discharge has been developed and that the forces generated can be very large and change rapidly. It is of major importance that these forces are taken into consideration when existing plants are modified or when new ones are designed.

In this paper it has been necessary to introduce a number of simplifying assumptions in order to make the calculations tractable. Currently a programme of fundamental work is being carried out with the objects of improving the methods described in this report. A computer program is being prepared to replace the graphical methods presented here; this will take account of entropy changes and pipe friction losses of total pressure will be determined from model and full scale tests to be carried out.

Acknowledgments

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Symbols Used

A = area.
$\mathbf{A}$ = area vector.
a = velocity of sound.
$@$ = reference velocity of sound (see text).
F = force.
$\mathbf{F}$ = force vector.
f = impulse function $(= \rho u^2 + p)\, A_p$.
k = isentropic exponent equal to the ratio:

$$\frac{\text{(specific heat at constant pressure)}}{\text{(specific heat at constant volume)}}.$$

L = reference length.
M = non-dimensional velocity (for shock wave = w/a_1).
p = static pressure.
R = gas constant.
T = temperature.
t = time.
$\mathbf{U}$ = velocity vector.
u = velocity of gas in pipe.
V = volume.
W = velocity of shock wave.
X = non-dimensional distance co-ordinate $(= x/L)$.
$\left.\begin{matrix} x \\ y \end{matrix}\right\}$ = Cartesian co-ordinates.
Z = non-dimensional time $(= @t/L)$.
θ = angle of deviation at a bend.
ρ = density.

Subscripts

$_A$ = atmospheric conditions.
$_B$ = on bend and behind shock wave.
$_C$ = on autoclave.
$_F$ = in front of shock wave.
$_I$ = inlet plane.
$_O$ = outlet plane.
$_P$ = pipe.
$_S$ = shock wave.
$_X$ = x-direction.
$_Y$ = y-direction.

$_1$ = low pressure condition in pipe before discharge. In the present work this is equal to the atmospheric pressure and is used as a reference pressure. $_2$ = condition behind moving shock wave.	Applies to gas in pipe on low-pressure side before rupture of disc.
$_3$ = condition in front of expansion wave. $_4$ = condition in autoclave before discharge.	Applies to gas on autoclave side of disc before discharge.

The above quantities may be expressed as any set of consistent units in which force and mass are not defined independently.

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APPENDIX

Details of a Typical Calculation for the Wave Action Period of Discharge from an Autoclave System.

Assumed conditions

The contents of the autoclave are a mixture of gases and for the purpose of calculation they are represented by a perfect gas with the following specification:

molecular weight = 16
gas constant = R_4 = 5570 ft lbf/slug deg K
$k_4 = 1{\cdot}1$
p_4 = 561 lbf/in^2 abs
T_4 = 1500°K
L = 28 ft.

TABLE I.—*Properties of States*

State	$\frac{a}{@}$	$\frac{u}{@}$	$\frac{u+a}{@}$	$\frac{u-a}{@}$	$\frac{P}{P_1}=\left(\frac{a}{@}\right)^{22}$	$1{\cdot}1\left(\frac{a}{@}\right)^{20}\left(\frac{u}{@}\right)^{2}$	$\frac{f}{A_pP_1}$
2		1·112			12·95	35·75	48·7
3	1·1234	1·112	2·235	0·011	12·95	13·9	26·9
4	1·1790	0	−1·179	−1·179	37·4	0	37·4
5	1·1284	1·009	2·137	−0·119			
6	1·1340	0·900	2·034	−0·234			
7	1·1410	0·758	1·899	−0·383	18·2	8·8	27·0
8	1·1482	0·613	1·761	−0·535	20·9	6·55	27·45
9	1·1578	0·425	1·583	−0·733	25·1	3·7	28·8
10	1·1663	0·254	1·420	−0·912	29·5	1·5	31·0
11	1·1322	1·287	2·419	+0·155	15·35	21·85	37·2
12	1·1373	1·185					
13	1·1426	1·074					
14	1·1497	0·932			21·5	15·5	37·0
15	1·1570	0·790			24·7	12·7	37·4
16	1·1665	0·598					
17	1·1750	0·428			34·8	5·1	39·9
18	1·1354	1·349	2·484	+0·214	16·3	25·4	41·7
19	1·1404	1·248					
20	1·1459	1·136					
21	1·1530	0·995			22·9	18·8	41·7
22	1·1601	0·852					
23	1·1696	0·663			31·3	11·6	42·9
24	1·1373	1·388	2·525	+0·251	17·0	27·9	44·9
25	1·1423	1·287			18·7	2605	44·75
26	1·1478	1·175			20·7	23·8	44·5
27	1·1549	1·034			23·75	20·95	44·7
28	1·1620	0·892			27·1	17·6	44·7
29	1·1380	0·138	2·539	+0·263	17·2	28·7	45·9
30	1·1430	1·300			18·9	26·95	45·85
31	1·1485	1·191			21·0	24·9	45·8
32	1·1556	1·050			24·0	21·8	45·7

TABLE II.—*Results for Autoclave System with Bend*

@ t (ft)	$f/A_pp_1 - 1$			Horizontal Force (tonf)				Time (ms)
	Autoclave	Bend	Deflector	F_{CX}	F_{BX}	F_D	$F_{CX}+F_{BX}-F_D$	
	36·4*	36·4*	0*	8·11	1·26	0	9·37	0
6·7	36·4*	36·4*	0*	8·11	1·26	0	9·37	2·60
8·7	36·4*	30·0*	0*	8·11	1·04	0	9·15	3·38
9·6	36·4*	28·9	0/47·7*	8·11	1·00	0–12·28	9.11-−3·17	3·72
10·9	36·4*	27·8*	47·7*	8·11	0·96	12·28	−3·21	4·23
11·9	36·4*	27·3	47·7*	8·11	0·94	12·28	−3·23	4.62
12·6	36·8	26·9	47·7/25·9*	8·20	0·93	12·28–6·67	−3·15–2.46	4.89
14·8	38·3	26·4*	25·9*	8·54	0·91	6·67	2·78	5·74
15·7	38·9*	29·1	25·9*	8·67	1·00	6·67	3·00	6.09
18·8	40·9	36·2*	25·9*	9·12	1·25	6·67	3·70	7.30
20·5	41·9*	38·3	25·9*	9·34	1·32	6·67	3·99	7·96
23·6	42·7	40·7*	25·9*	9·52	1·40	6·67	4·25	9·16
26·0	43·0	41·8	25·9*	9·58	1·44	6·67	4·35	10·09
28·5	43·3	42·5	36·2*	9·65	1·47	9·31	1·81	11·07
32·5	43·6	43·3	40·7*	9·71	1·49	10·48	0·72	12·61
33·1	43·7*	43·3	41·1	9·75	1·49	10·59	0·65	12·84
35·8	43·8	43·6*	42·1	9·76	1·50	10·84	0·42	13·90
44·8	44·2	44·1	43·9*	9·85	1·52	11·30	0·07	17·4
73·2	44·7*	44·75*	44·5	9·96	1·54	11·46	0·04	28·4
85			44·9*			11·58		

* Values of $f/A_pP_1 - 1$ marked by an asterisk are from known states on the physical plane diagram. Other values are interpolations.

Two values [illegible] shown where sudden changes occur.

The pipe downstream of the disc contains air for which the following are assumed:

molecular weight = 29

gas constant = R_1 = 3105 ft lbf/slug deg K

$k_1 = 1{\cdot}4$

$p_1 = 15$ lbf/in² abs

$T_1 = 298°$K

The discharge pipe is 7 in. diameter.

Construction of the characteristic diagrams—Figs 2 and 3

STATE DIAGRAM

The parameters used for the state diagram are $u/@$ and $a/@$ where @ is a reference speed of sound which is related to a_4 and p_4/p_1 by the equation:

$$\frac{a_4}{@} = \left(\frac{p_4}{p_1}\right)^{(k_4-1)/2k_4}.$$

The non-dimensional form of equation (5) is:

$$\frac{da/@}{du/@} = \mp\left(\frac{k-1}{2}\right) = \mp\tfrac{1}{20}.$$

POSITION DIAGRAM

The parameters used for the position diagram are $Z = @t/L$ and $X = x/L$ and the non-dimensional form of equation (4) is:

$$\frac{dX}{dZ} = \frac{u}{@} \pm \frac{a}{@}.$$

In this case $L = 28$ ft.

STEADY FLOW ENERGY EQUATION

Equation (6) in non-dimensional terms becomes:

$$\left(\frac{a_4}{@}\right)^2 = \left(\frac{a}{@}\right)^2 + \left(\frac{k-1}{2}\right)\left(\frac{u}{@}\right)^2$$

which becomes:

$$\left(\frac{a}{@}\right)^2 + \tfrac{1}{20}\left(\frac{u}{@}\right)^2 = (1{\cdot}179)^2.$$

SPEED OF SHOCK WAVE AND CONTACT SURFACE

$$\frac{a_1}{a_4} = \sqrt{\left(\frac{k_1 R_1 T_1}{k_4 R_4 T_4}\right)} = 0{\cdot}374$$

$$\frac{p_4}{p_1} = 37{\cdot}4.$$

Hence from equation (1):

$$\frac{p_2}{p_1} = 12{\cdot}95$$

and equation (2):

$$M_s = \frac{W}{a_1} = 3{\cdot}36$$

The referece speed of sound, @, is related to the speed of sound at release, a_4, by equation (7):

$$\frac{a_4}{@} = \left(\frac{p_4}{p_1}\right)^{(k_4-1)/2k_4} = 1{\cdot}179.$$

Hence:

$$\frac{a_1}{@} = \left(\frac{a_1}{a_4}\right)\left(\frac{a_4}{@}\right) = 0{\cdot}374 \times 1{\cdot}179 = 0{\cdot}441$$

$$\frac{W}{@} = \frac{W}{a_1}\,\frac{a_1}{@} = 1{\cdot}480.$$

The speed of the contact surface us $u_2 = u_3$ using equation (3) and the above expressions for a_1/a_4, $a_4/@$, or from the state diagram we can obtain:

$$\frac{u_2}{@} = \frac{u_3}{@} = 1{\cdot}112.$$

The construction of the characteristic diagrams now follows the usual procedures, *e.g.* those given by Shapiro.[4] The states at the mesh points are given with other information in Table I.

Construction of the graphs of force against time

GAS FROM THE AUTOCLAVE

The flow of the gas which was originally on the high-pressure side of the disc takes place at constant entropy and in this case the impulse function was expressed in equation (13) in terms of the parameters used on the state diagram. In this case it becomes:

$$\frac{f}{A_p p_1} = \left(\frac{a}{@}\right)^{22} + 1{\cdot}1\left(\frac{a}{@}\right)^{20}\left(\frac{u}{@}\right)^2.$$

Values of this impulse function are also given in Table I.

GAS ORIGINALLY IN THE PIPE

During the discharge period when region 2 in Fig. 2 is at the boundary, the gas flowing from the pipe is the gas which was originally in the pipe rather than the gas from the autoclave. In general these gases are different and isentropic relationships developed above do not apply. In this case:

$$\frac{f}{A_p p_1} = \frac{\rho_2 u_2^2}{p_1} + \frac{p_2}{p_1}$$

and the pressure term is evaluated from:

$$\frac{p_2}{p_1} = \frac{p_3}{p_1} = \left(\frac{a_3}{@}\right)^{22}.$$

The first term on the right-hand side is evaluated as follows:

$$\frac{\rho_2 u_2^2}{p_1} = k_1\left(\frac{\rho_2}{\rho_1}\right)\left(\frac{@}{a_1}\right)^2\left(\frac{u_2}{@}\right)^2.$$

The density ratio is given by the well-known Rankine–Hugoniot equation:[4]

$$\frac{\rho_2}{\rho_1} = \left[\frac{\left(\frac{k_1+1}{k_1-1}\right)\frac{p_2}{p_1}+1}{\frac{p_2}{p_1}+\left(\frac{k_1+1}{k_1-1}\right)}\right]$$

$$\frac{u_2}{@} = \frac{u_3}{@}\quad\frac{@}{a_1} = \frac{1}{0{\cdot}441}\ \text{(derived above).}$$

We obtain:

$$\frac{\rho_2 u_2^2}{p_1} = 35{\cdot}7.$$

AUTOCLAVE BOUNDARY

The gas force in the autoclave is:

$$\sqrt{(F_{CX}^2+F_{CY}^2)} = \left(\frac{f_c}{A_p\,p_1} - 1\right) A_p\,p_1$$

and the horizontal component is:

$$F_{CX} = \left(\frac{f_C}{A_p\,p_1} - 1\right) \cos\theta\, A_p\,p_1$$

$$= \left(\frac{f_c}{A_p\,p_1} - 1\right) 0{\cdot}866\, A_p\,p_1.$$

BEND

The gas force on the bend in the horizontal direction assuming the inlet and outlet pressures, densities and velocities are equal is:

$$F_{BX} = \left(\frac{f_B}{A_p\,p_1} - 1\right)(1-\cos\theta)\, A_p\,p_1$$

$$F_{BX} = \left(\frac{f_B}{A_p\,p_1} - 1\right) 0{\cdot}134\, A_p\,p_1.$$

DEFLECTOR

The expression for the gas force on the deflector, Fig. 7, is calculated from Fig. 4B:

$$F_D = A_p\,\rho u^2 + A_p(p-p_1)$$

hence:

$$\frac{F_D}{A_p\,p_1} = \left(\frac{\rho u^2}{p_1} + \frac{p}{p_1} - 1\right) = \frac{(f_D-1)}{A_p\,p_1}$$

this is also given in Table II.

The net gas force on the autoclave system is therefore

$$(F_{CX}+F_{BX}-F_D)$$

These values are given in Table II.

The manuscript of this paper was received on 1 *April*, 1967.

DISCUSSION

Mr. D. d. OWEN said that he was engaged on a project, in co-operation with the authors, in extending the work reported in the paper. Currently, preliminary tests were being carried out to prove the instrumentation to be used for a detailed investigation on a model autoclave, approximately one quarter full size.

A full range of tests were to be carried out both on the model autoclave and on a full-scale autoclave.

It was proposed to measure the transient forces and pressures which exist in the system, at any particular stage in the discharge process.

The theoretical work was concerned with the method of characteristics and was directed towards the preparation of a computer programme. The programme being prepared would involve a comprehensive treatment, and it was hoped that it will ultimately be used as an aid to design work.

Later it was intended to study the discharge process when the fluid expelled entrained small solid particles.

Mr. F. MARSHALL asked for an explanation of the significance of the axial position of the bursting disc along the discharge pipe. If the bursting disc were to be positioned directly on to the autoclave how would it affect the forces? He assumed that the sizing of the bursting disc was consistent with the design strength of the autoclave.

Mr. THORNTON replied that the actual position of the disc was determined by a number of factors which included the ease of replacement and the prevention of solids and liquid lodging on top of the disc. A change in the location of the disc would change the wave action; for example, if the disc were moved towards the autoclave, the point C on 2 and 8 would occur sooner and points B and D later. Point F would also occur earlier and eventually the reflected wave 3–11–8 *etc.* (2) would interact with the shock wave making it stronger.

In the limit, when the disc was placed at the autoclave, the shock wave would be stronger than in the example given and the force on the autoclave would rise more rapidly.

Finally, the size of the disc was determined by considerations such as the time rate of increase of pressure, the specified bursting pressure, and the capacity of the container to be vented.

Mr. J. R. CROWTHER said that B.P. had a number of rocking autoclaves in operation. They were of 2 litres capacity and bursting disc rating of 4000 lbf/in^2. In this case the bursting disc was always put on the head of the vessel. It was not situated at some distance down the pipe. It was not safe to put in flexible high pressure tubing to connect to some distance down the discharge stack. That introduced a complication in so far as allowance had to be made for the temperature of the disc. If one had a reaction vessel at 300°C the temperature was 100 to 150°C at the position of the disc, and a temperature allowance had to be made on the rating of the disc to account for that. Experimental work had to be done to determine that allowance.

The paper interested him very much since it was the first time he had seen the momentum theorem used to calculate the forces involved in a disc failure.

With regard to rain, a complete cap would be put over the discharge pipe with a total 180° deflection on the pipe. Unless that were done, there would be about half a pint of water sitting on top of the disc. In the Middlesex area, rainwater was not just water, it was dilute sulphuric acid and the discs suffered. Allowance had to be made for the fact that that doubled the force on the deflector.

Mr. THORNTON and Dr. WOODS were interested in Mr. Crowther's remarks about the small high pressure autoclaves and the importance of the temperature of the disc in relation to its bursting pressure. Mr. Thornton added that in the paper the pipe containing the bursting disc was horizontal and therefore rainwater could not collect at the disc.

Mr. M. KNEALE said that his company had a number of situations where they had designed bursting disc systems and he shuddered to think of the amount of work he now had to do to see that they were safe. He wondered whether the remarks the authors had made applied to very high pressure systems only. As an example, if one considered a system at 100 lbf/in^2 with a bursting disc of 6 in. diam mounted directly on the vessel, and it blew, would the forces generated under such a condition be very serious?

Mr. THORNTON replied that the methods of calculation were applicable to both high and low pressures, within the range of validity of the assumptions such as perfect gases, *etc.*

The question of whether the forces generated are serious, depends upon the pipework geometry and method of support. However, with a 6 in. diam pipe and a pressure of 100 lbf/in^2 absolute, forces of up to 3700 pounds force could be expected.

Dr. W. E. MASON asked if the authors could indicate what percentage of the contents of the autoclave was discharged during the wave action period in the examples considered.

Mr. THORNTON replied that calculations were based on the assumption that during the wave action period conditions in the autoclave did not change; that implied that a negligible proportion of the contents was discharged in this time. However for the calculation given in the paper, it was expected that the mass discharged in the wave action time (about fifteen milliseconds) would be less than 1% of the total mass to be discharged. The above assumptions therefore seemed reasonable.

Mr. R. J. KINGSLEY said that it had been assumed that the rupture was instantaneous. He wondered how significant that was in relation to the treatment and what difference there would be if, instead of the rupture disc, there was either a safety valve or perhaps even a valve operated by some deliberate mechanism and opening over a time span of, say, fifteen seconds.

Dr. WOODS said that the distance–time diagrams for instantaneous rupture were drawn from a point and an expansion wave and shock wave generated immediately. If the rupture took place over a period of time, a pressure wave would be formed during this interval which would develop into a shock wave later, and the expansion wave would not be a centred expansion wave. There were techniques for dealing with this theoretically.

He thought that a safety valve with an opening period of fifteen seconds would give quasi-steady flow almost from the start. As Mr. Thornton had said, the wave action period was all over after about fifteen milliseconds.

Dr. C. R. BLACK asked whether the theory could be applied to gases where there was an explosion in a vessel which caused a disc to burst as the pressure was still rising. Could an impression be incorporated for the rate of pressure rise?

Mr. THORNTON said that the method could be used even when the conditions in the autoclave were changing with time. It would involve using a succession of boundary curves each one being applicable to a given instant of time. A sufficient number of boundary curves would have to be used to give small changes in boundary conditions and the number involved would therefore depend upon the rate of change of conditions in the autoclave.

However, it should be pointed out that information on the time history of the explosion would be needed and caution should be exercised in this kind of application.

Dr. WOODS said that the calculations might appear laborious but a research student was working on a computer program to do this. The work had developed from other unsteady flow problems connected with engines. In an engine not only were the conditions in the discharge vessel (*i.e.* engine cylinder) changing with time, but the valve area and cylinder volume also changed with time.