AN ACOUSTIC MODEL FOR PREDICTING THE OVERPRESSURES CAUSED BY THE DEFLAGRATION OF A GROUND LYING VAPOUR CLOUD.

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A mathematical model of the overpressure field due to the deflagration of a point ignited ground lying vapour cloud is described. The model is based on acoustic theory with additional assumptions about the changes in the cloud shape, flame speed and flame shape based on experimental observations.

Close agreement has been obtained between the model and soap bubble experiments performed by ENSMA on centrally ignited flat circular clouds. The model is then used to compare the overpressures due to interior and edge ignition of flat topped clouds for different cloud heights and base shapes.

INTRODUCTION

This paper considers the explosion of a flat vapour cloud whose height is much smaller than its base dimensions. This is typical of the cloud that would form in the event of a large release of refrigerated fuel gas. The results only strictly apply to unobstructed terrain since they omit the effects of flame acceleration and partial confinement caused by buildings and plant. However the high burning velocity assumed in the calculations could only be achieved in the presence of turbulence promoting obstacles. In this case therefore the overpressure predictions should be interpreted as that due to the mean flame propagation excluding localised effects which may arise when a flame interacts with an obstacle.

As the vapour cloud disperses, possibly aided by wind, it can assume a variety of base shapes. It may then ignite at its boundary or in its interior depending upon whether the ignition source which it engulfs is continuous or intermittent. All these variables are investigated.

The model described in this paper takes account of the important physical processes occurring in the deflagration explosion which have been observed in experiment(1). The vapour cloud's burning properties will be assumed uniform.

Interior ignition gives rise to the highest flame speeds because all the combustion products are enclosed by the hemi-spherical flame formed. This speed is maintained up until the flame reaches the nearest boundary of the vapour cloud, which will be pushed outwards by the piston action of the flame. For the purposes of this paper an interior ignition point will be assumed well inside the fuel boundary and hence the hemi-spherical flame will always partially extinguish first on the cloud roof. The flame

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extinction provides an escape route for the combustion products which is accompanied by a pressure relief wave which propagates outward from the flame. The flame will then decelerate as the product relief area grows and also as a result of the backward pull exerted by the buoyant product gases. The resulting cylindrical flame will have some curvature and the flow immediately ahead of it will distort the fuel cloud, making the flame height greater than the initial height of the cloud. This phase will be referred to as the cylindrical phase of flame propagation.

For edge ignition the flame will always have an approximately part-cylindrical shape since the combustion products can escape. The flame speeds and the overpressures generated at the flame must therefore be less than for an interior ignition.

The model employs an acoustic solution to the approximate linearised equations of compressible gas motion which simplifies the overpressure calculation. It has been shown(2) that such an approach gives sufficiently accurate overpressure predictions for a hemi-spherical flame at flame speeds up to half the ambient sound speed. This is true apart from very close to the flame, where the flow is better approximated as incompressible. In the cylindrical phase the flame is modelled as a distribution of acoustic sources, each moving with the flame. Since the flame speed and flame shape seriously affect the accuracy of the predictions, the modelling assumptions to take account of flow distortion of the vapour cloud immediately ahead of the flame and the changes in flame speed are therefore described in detail in the following sections.

Finally, the model predictions are used to compare with experimental data and to evaluate the effects of varying the ignition position and vapour cloud shape and size.

THE DESCRIPTION OF THE MODEL

The Overpressure due to the Hemi-spherical Flame

A good approximation to the point source acoustic solution for the over-pressure due to a growing hemi-spherical flame with burning velocity Su is well known(3) and is given by

$$\Delta P/P_0=2\gamma E^2(E-1)(Su/a_0)^2(a_0t-R)/R;\ o\leq a_0t-R$$

The pressure relief caused by partial flame extinction on the cloud roof is approximated by that due to complete flame extinction. Finite Difference calculations of this are shown in Fig. 1(a) showing a negative pressure wave which is adequately approximated by

$$\Delta P/P_0=2\gamma E^2(E-1)(Su/a_0)^2(a_0(t-2\Delta t)-R)/R;a_0\Delta t\leq a_0t-R\leq 2a_0\Delta t$$

where Δt is the apparent time from ignition to flame extinction seen by an observer.

Vapour Cloud Displacement by the Flame

The hemi-spherical flame grows to a radius of $E^{\frac{1}{3}}h_0$ before partially extinguishing on the cloud roof. Assuming incompressible flow ahead of it then a point at radius r_0 from the ignition point moves to r_1 as Fig. 1(b) where

$$r_1^3 = r_0^3 + (E-1)h_0^3$$

The initial cloud height is adjusted according to this formula.

The flow ahead of the flame in its cylindrical phase will also cause the flame height to be above the initial cloud height(1) and will in addition increase the base area of the initial cloud. This will depend upon the flame curvature and is difficult to model precisely. The model therefore increases the initial cloud height by some constant factor to take account of this, necessarily less than $E^{\frac{1}{3}} h_0$, and allows for any increase in the base area in the initial cloud shape.

The Over-Pressure due to the Cylindrical Flame

A number of authors have applied the acoustic approximation to model the cylindrical phase of flame propagation, some by idealising the explosion source as a point(3), others by a line of acoustic sources along the base of the flame(4). In all, however, the flame is assumed to be that part of a circular cylinder centred on the ignition point lying within the initial vapour cloud and the cloud is assumed not to move during the explosion. These assumptions result in unphysical effects as described below.

The overpressure due to an element of flame area dA_F is approximately proportional(3) to its rate of change in time $d\dot{A}_F$. The flame overpressure therefore splits into two contributions, a positive one due to the bulk of flame which is growing in area and a negative one originating from the flame edges due to the destruction of flame area. The negative contribution is of a similar magnitude to the positive one and is very sensitive to the assumed shapes of the flame and fuel cloud.

When the flame edge contributions are included, as is usual in this type of model, the overpressure is predicted to become negative and unbounded at the end of an edge ignited circular pancake cloud explosion(4). This is also the reason why the overpressure due to an end ignited cigar shaped cloud goes negative as the flame passes the widest part of the cloud(5).

These results are questionable physically. In practice the flow field will distort the flame and its speed will be lower near the cloud boundary reducing the rate of flame extinction.

In view of this a different approach has been taken here to modelling the flame in the cylindrical phase by omitting the sources of negative overpressure at the flame edge. The positive overpressure remaining is however very dependent upon the assumed local rate of growth of the flame area. On physical grounds the flame can be expected to take on some curvature and so for the purposes of this model, it is assumed to be locally spherical with radius of curvature equal to the distance from the ignition point to the given point on the flame.

The overpressure is calculated by summing together the individual contributions of elements of flame area dA_F. The effect of the ground is taken into account by including the image of each element as if reflected in the ground plane. Each element is modelled as a simple point acoustic source with strength (E-1) Su dA_F moving outward with constant flame velocity S_F along a radial line from the ignition point. It can be shown(6) that the infinitessimal contribution to overpressure, d(ΔP), which this produces is given by

$$d(\Delta P)/P_0 = \frac{\gamma(E-1)\ Su}{4\pi a_0^2}\left[\frac{d\mathring{A}_F\,|_{t-R/a_0}}{R(1+S_F R'/a_0)} - \frac{S_F(S_F/a_0+R')dA_F\,|_{t-R/a_0}}{R^2(1+S_F R'/a_0)^3}\right]$$

where dA_F and $d\mathring{A}_F$ are evaluated at the retarded time $t - R/a_0$ and R' is the rate of change of the observer/element displacement with respect to the flame radius r_F for fixed Θ and μ (see Fig. 2). The assumptions on flame curvature give

$$dA_F = r_F^2 d\Theta d\mu \ ; \ d\mathring{A}_F = 2r_F S_F d\Theta d\mu$$

The above overpressure calculation is only valid outside the combustion zone and because of the inaccuracy of the acoustic approximation near the flame over-estimates the overpressure at the flame. The flame overpressure is therefore calculated using the ideas of the following section.

Asymptotic Flame Speed Estimation

Whether due to edge ignition or following partial flame extinction on the cloud roof from an interior ignition the flame in the cylindrical phase of combustion will rapidly achieve a limiting or asymptotic speed which can be expected to coincide with zero overpressure behind the flame. It follows from conservation of mass and momentum across the flame that the flame overpressure is directly related to the burning velocity through

$$\Delta P/P_0 = \gamma\ (E-1)\ (Su/a_0)^2$$

In addition, the acoustic model of the flame provides a relationship between the flame speed and the pressure ahead of the flame which in addition to the above equation relates the asymptotic flame speed to the burning velocity.

Overpressures due to a cylindrical flame of large radius were calculated using the acoustic model. Because the acoustic model overestimates the flame overpressure the accuracy of the calculation was improved (2) by asymptotically matching an incompressible solution to the acoustic one as Fig 3(a). The incompressible flow field immediately in front of the flame was assumed to be cylindrical, the flame having a radius of curvature of three initial cloud heights in the vertical plane (as estimated from experiment (1)) and an infinite radius of curvature in the horizontal plane. The predicted Asymptotic Flame Speed/Burning Velocity relationship is given in Fig. 3(b) for flame speeds within the range of validity of the acoustic theory.

COMPARISON WITH EXPERIMENT

Leyer(1) discusses some very small scale experiments using a flat circular cloud of radius 22cm which was confined within a soap membrane attached to a wire ring. The experimental results are for clouds with three different aspect ratios (ε= cloud height/base radius) each centrally ignited as Fig. 4. The final large surge in overpressure in each experiment was due to the flame's interaction with the wire ring. The model, which assumes no flame/obstacle interaction, has been used to predict the overpressures up to the flame's interaction with the ring.

The experiments were performed on an elevated circular table of radius 85cm with the pressure recordings taken 50cm from the table edge. The over-pressure field however does not correspond to a fully ground reflected one because the table has a finite radius. This is contrary to the modelling assumptions made by Leyer.

Although the pressure wave exhibits the effects of ground reflection when on the table there is a pressure drop as it spills over the edge. The expansion wave this produces propagates back to the microphone and is registered as a sudden change in the slope of the pressure/time recording at approximately 3 msec after the microphone first registers the arrival of the pressure wave. This can be seen in the original pressure/time recordings(1). Very little of the effects of ground reflection are present in this experiment and the over-pressures generated therefore are close to half of those predicted by the ground reflected theory.

Constant flame speeds of 5 m/s and 3.5 m/s, as estimated from Schlieren films, were used to model the hemi-spherical and cylindrical phases of flame propagation respectively. The asymptotic flame speed theory was not applied in this case since the actual speed is known. An expansion ratio of 8 was used for the near stoichiometric Ethylene/air mixture and the flame height during the cylindrical phase was assumed to be twice the initial cloud height, again as seen on the Schlieren film.

The model gives very close agreement to the measured overpressures as shown in Fig. 4 especially so for the lower aspect ratio clouds of 0.2 and 0.1. The over-pressures are in excess of the theory for the 0.29 aspect ratio cloud but this could be due to the turbulence generated by the soap bubble bursting. However, the cylindrical flame speed of 3.5 m/s is nearly twice that predicted by the asymptotic flame speed theory. This could be due to a sudden increase in burning velocity following the hemi-spherical phase. The flame speeds indicate the initial burning velocity was very near laminar in which case the perturbations caused by the cessation of the hemi-spherical phase could have induced flame instabilities which would explain a doubling in burning velocity.

PREDICTIONS OF THE EFFECT OF CLOUD SHAPE AND IGNITION POSITION

The following calculations were done using a very high burning velocity of 20 m/s and an expansion ratio of 8 making the hemi-spherical overpressure calculation at the very extreme of its range of validity. The flame height was taken to be twice the initial cloud height during the cylindrical phase of combustion as exhibited in the experiment of Leyer described above.

Overpressure Dependence on Aspect Ratio

Predictions of the decay in peak overpressure with distance from centrally ignited flat topped circular clouds with aspect ratios of $^1/_{16}$,

$^1/_{32}$ and $^1/_{64}$ are shown in Fig. 5 compared against that for a hemi-spherical cloud of height h_0. Near to the edge of a flat cloud, within approximately one cloud radius, the cylindrical phase overpressures dominate. In the far field the overpressure pulse resulting from the hemi-spherical phase of combustion dominates over the more rapidly decaying cylindrical phase over-pressure.

Overpressure Dependence on Cloud Shape

Predictions of the overpressure decay along the major and minor axis directions for a centrally ignited elliptical cloud are as Fig. 6. The major axis radius was taken as 64 times the cloud height and 4 times the minor axis radius. The pressure/time insets to Fig. 6 show the pressure pulse due to the hemi-spherical phase to be equal, at a distance of $80h_0$ from the ignition point, along both major and minor axes. The overpressure field due to the cylindrical flame on the other hand is not symmetric with smaller pressures occurring along the minor axis direction. This is due to the flame extinguishing further away from an observer position on the minor axis.

Comparison Between Central and Edge Ignition

Predictions of peak overpressure decay with distance for three different ignition positions of a circular, flat topped cloud with an aspect ratio of $^1/_{64}$ are given in Fig. 7. The overpressures are little different for the two cases of interior ignition and at a sufficient distance from the cloud the two merge. On the other hand the edge ignition case, without the hemi-spherical phase, produces significantly lower overpressures. The overpressure field also ceases to be symmetric about the cloud in the far field with the lowest pressures recorded by observers who see the flame travelling away from them as Fig. 8.

CONCLUSIONS

In this paper the essential components of an acoustic model of a ground lying vapour cloud explosion have been described. The model has been partially validated using the measured average flame speeds by obtaining close agreement with the pressures measured in soap bubble experiments using centrally ignited flat circular clouds. These very small scale experiments however do not provide an adequate test of the asymptotic flame speed theory.

The model has also been used to derive some important conclusions concerning unobstructed flat cloud combustion. The far field overpressures due to interior point ignition of a flat cloud irrespective of its base shape, are dominated by the overpressures due to the hemi-spherical phase of combustion. For a given vapour and burning velocity, therefore, the far field overpressure is proportional to the cloud height above the ignition point and not related to the total volume of vapour in the cloud. As the cloud edge is approached, within approximately one cloud radius of its edge for the low aspect ratio clouds considered, the cylindrical phase overpressures dominate. For an elongated cloud the highest and lowest cylindrical phase overpressures occur along the major and minor axes of the cloud respectively.

The overpressures due to an edge ignited cloud, which is the more probable situation for a vapour cloud being dispersed by wind, are significantly less than for interior ignition. For an edge ignition the overpressure field is asymmetric with the highest and lowest overpressures being experienced respectively by observers towards which and away from which the flame is travelling.

Acknowledgement

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NOMENCLATURE

A_F	Flame area
a_0	Ambient sound speed
dA_F	Element of flame area
$d\mathring{A}_F$	Rate of change of the area of the flame element in time as seen by an observer travelling with the flame.
$d(\Delta P)$	Over-pressure due to flame area element dA_F
E	Expansion ratio
h_0	Initial cloud height above ignition point
P_0	Absolute pressure
R	Distance from acoustic source to observer
r	Radial distance from ignition point
r_E	Flame radius at the time of partial extinction on the cloud roof
r_F	Flame radius
S_F	Flame speed relative to the ground
Su	Burning velocity; flame speed relative to the unburnt gas immediately ahead of it
t	time

Greek Symbols

ε	Ratio of cloud height to base radius.
ΔP	Over-pressure = P - Po
Δt	Apparent duration of hemi-spherical phase of flame propagation as seen by an observer = $r_E/a_0(a_0/S_F-1)$

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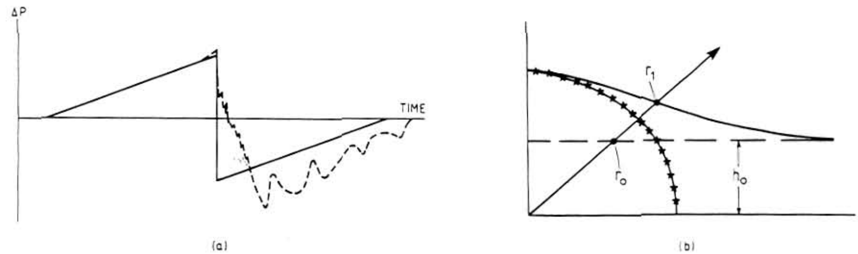
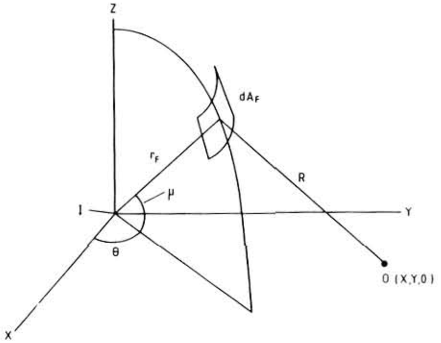


Fig. 1. (a) Pressure wave due to the constant flame speed deflagration of a point centrally ignited hemispherical cloud; -----, Finite Difference calculation; ________, acoustic approximation. (b) Displacement of vapour cloud by hemi-spherical flame; -x-x-x, flame; __ __ __, initial cloud, _____ at end of hemi-spherical phase.



I, ignition point;

O, observer position

Fig. 2 Cylindrical flame co-ordinate system

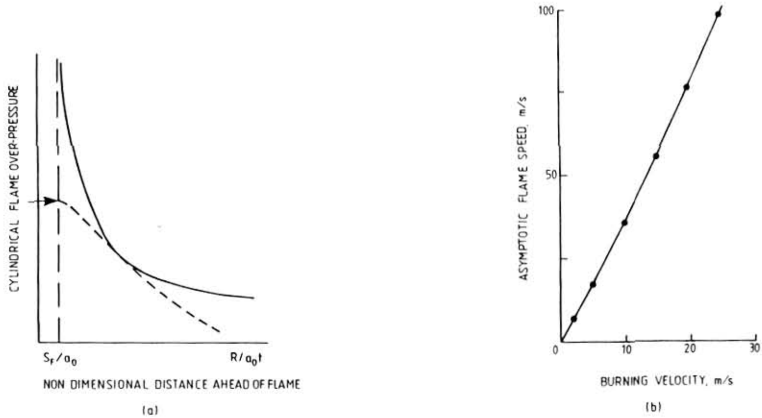


Fig. 3 Asymptotic flame speed estimation. (a) Asymptotic matching of acoustic and incompressible solutions to calculate over-pressure at the flame; ____, acoustic solution; ----, incompressible solution.
(b) Calculated relationship between asymptotic flame speed and burning velocity.

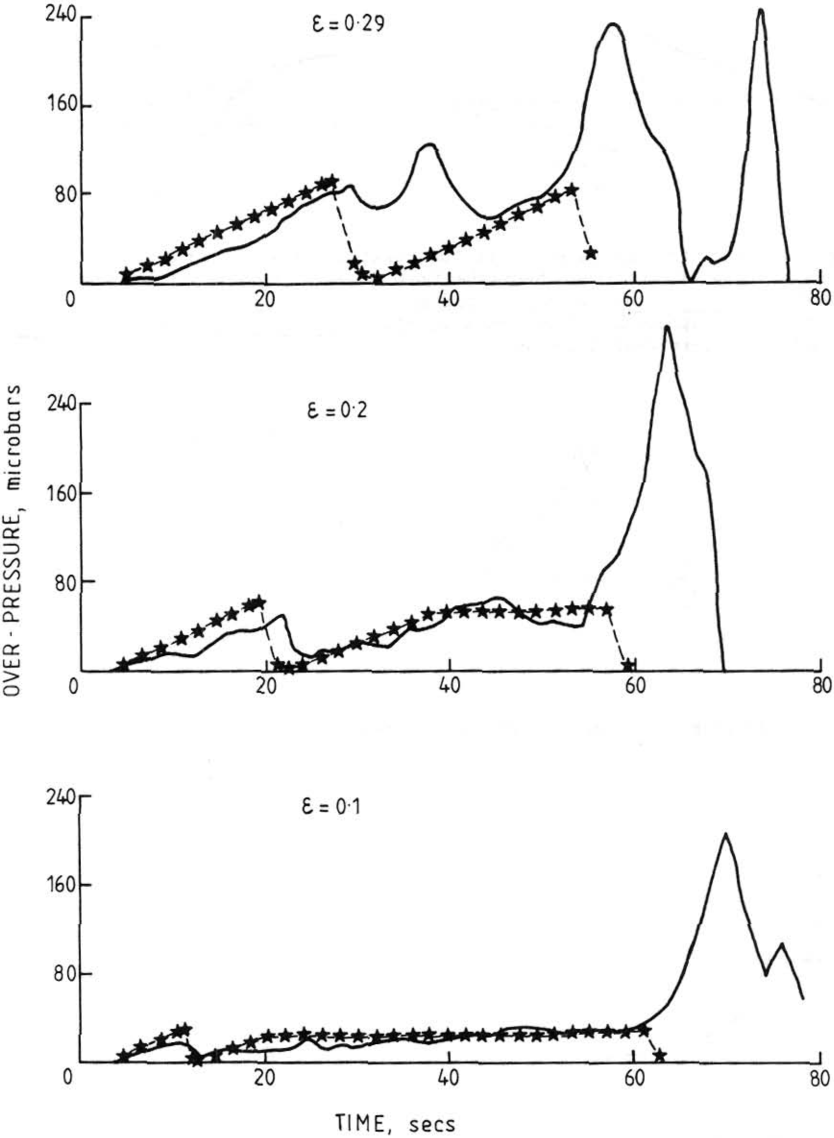


Fig. 4 Comparison between the model predictions and experimentally measured overpressures; ____, experiment; ++++, model predictions assuming 5.0 m/s hemi-spherical and 3.5 m/s cylindrical flame speeds.

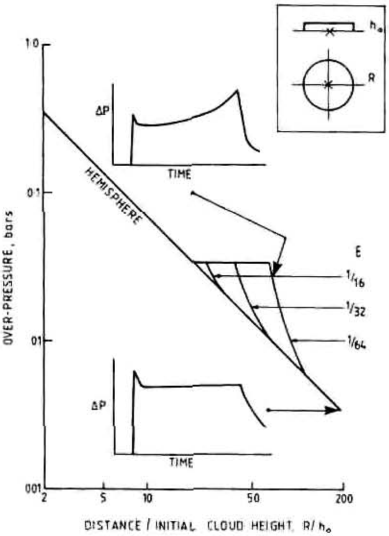


Fig. 5 Relationship between overpressure and normalised distance against initial cloud height for a hemi-spherical cloud and three flat clouds, all centrally ignited, with aspect ratios as shown in the inset. The insets show the differences in overpressure versus time in positions where the cylindrical and hemi-spherical phase overpressures dominate for a flat cloud.

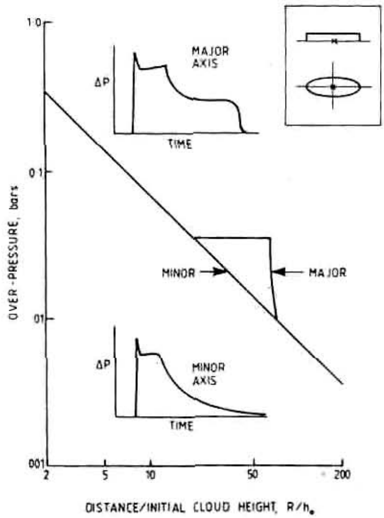


Fig. 6 Relationship between overpressure and distance for a flat elliptical cloud as shown in the inset. Major axis radius = 64 h_0, minor axis radius = 16 h_0. The insets show overpressure time at a distance of 80 h_0 on both major and minor axes.

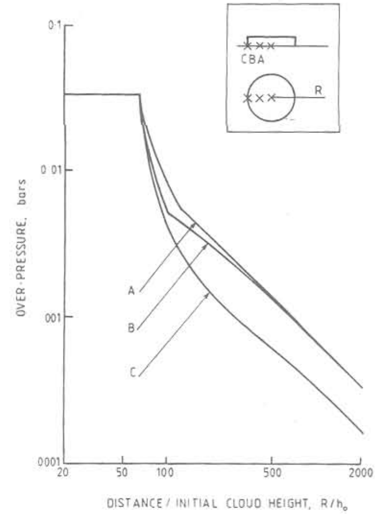


Fig. 7 Relationship between overpressure and distance measured in the direction as indicated in the inset due to interior and edge ignition of a flat circular cloud with aspect ratio $^1/_{64}$.

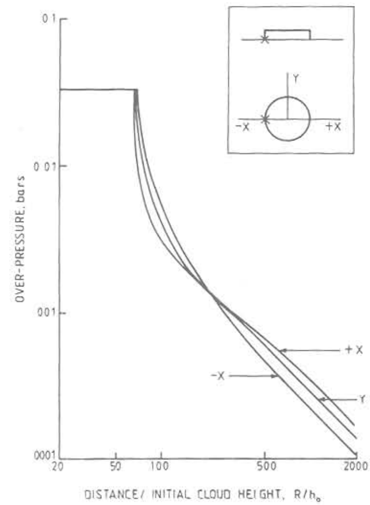


Fig. 8 Relationship between overpressure and distance along three mutually perpendicular directions about an edge ignited flat circular cloud with aspect ratio $^1/_{64}$.