Assessing Personnel Risk Associated with Projectiles resulted from Over Pressure Failures

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It is well known that the failure of a pressurized system may result in the creation of a blast wave. The industry has well established methods for determining the probability of fatality inside and outside buildings as a function of distance resulting from the blast wave. Projectiles may also be generated directly and indirectly from a blast wave and these can have serious consequences. Methods exist to predict trajectories and probability of fatality if hit, but no method is currently available that predicts the probability of impact as a function of distance. Previous work focused on predicting the distribution of projectiles on the ground, based on real events and theoretical calculations. Such methods are subject to many uncertainties and do not determine an overall probability of impact.

Here a new method is presented that directly assesses the risk to personnel from projectiles. This method takes a different approach, focusing on the probability of hitting a vertical target, such as a person, as a function of distance. The analysis of the problem results in a surprisingly simple and robust method for determining the probability of impact. The method is based on the extension of information contained in well-established references.

The simple model for predicting probability that a projectile will hit a vertical target is shown to be consistent with more rigorous statistical analysis and is easily extended to multiple projectiles to predict overall probability of fatality.

The new methodology is suitable for a simple spreadsheet based tool to allow easy use for risk assessments of vessel failures associated with a pressure volume (PV) energy release.

Introduction

Failure of pressurized systems may result in the creation of blast waves. These blast waves can produce significant damage to infrastructure, buildings and personnel inside or outside buildings. There have been many studies on the quantification of blast waves and the impact they have on structures (Saville 1997, Alonso 2006, Haque 2009, CCPS 2010). Also, well documented is the quantification of fragment distribution and size (Baker 1983, McCleskey 1988) and effect of impact on the human body (Baker 1983, Barbet et al. 1996, Cole 1997, Patel 2012). In a study of fatalities from industrial incidents producing blast waves (Settles 1968), of the eighty-one incidents examined there were seventy-eight fatalities. Seventy-seven of the fatalities were from radiant heat or projectile impact. One fatality was due to the blast wave causing bodily displacement and deceleration. However, there is no readily available assessment methodology to determine the probability of impact from projectiles resulting from an explosion.

The objective of this work has been to produce a methodology that will predict vulnerability of personnel to projectiles resulting from vessel and pipe failure. This will compliment methodologies which address blast wave, thermal radiation and other loss of containment events, and so complete a full vulnerability risk assessment.

All symbols are defined in the Nomenclature section.

Probability of Impact on a Target

Projectiles from the failure of a pressurized system will have a range of sizes and shapes, and fly in any direction, at any angle of projection and a range of initial velocities. To start the analysis, we considered how a single small projectile, with no drag, could hit a vertical target in the field, see Figure I. The initial launch angle, \( \alpha \), required to hit the base of the target can take two values, a low value for a flat trajectory and higher value for a howitzer style or lob trajectory. Similarly, to hit the top of the target there are two more angles, one low and one high. The target will be hit if the initial trajectory angle is:

\[
\alpha_1 < \alpha < \alpha_2 \quad \text{or} \quad \alpha_3 < \alpha < \alpha_4
\]

Assuming that the projectile can travel in any direction, from straight up in the air to straight down to the ground, \(-\pi/2 < \alpha < \pi/2\), then the probability of impact on the target, in the x-y dimension, is given by:

\[
P_{xy} \left( \frac{\alpha_2 - \alpha_1}{\pi} \right) \left( \frac{\alpha_4 - \alpha_3}{\pi} \right)
\]

To find the values of \( \alpha \) required to hit that target, we start with the trajectory equations, with no drag term:

\[
\frac{dy}{dt} = V_0 \sin(\alpha) - g \cdot t \quad \text{and} \quad \frac{dx}{dt} = V_0 \cos(\alpha)
\]

Solving for \( x \) and \( y \), then eliminating \( t \), we get a quadratic equation in \( \tan(\alpha) \), terms of initial velocity and position:

\[
\left( \frac{g x^2}{2 V_0^2} \right) \tan^2(\alpha) - x \tan(\alpha) + \left( \frac{g x^2}{2 V_0^2} + (y - y_0) \right) = 0
\]

Solving equation 4 for each value of: \( y = 0 \), the base of the target, and, \( y = H \), the top of the target, gives the four values of \( \alpha \) needed to calculate the probability of impact in equation 2, as a function of target height, \( H \), and position, \( x \).
The overall probability of impact needs to consider the radial distribution of projectiles and the width of the target, W. The projectiles can go in any direction around the circle, 2π, as seen in Figure 2. So, the probability of radial impact is given by:

\[ P_r = \frac{\beta}{2\pi} \quad \text{where:} \quad \tan\left(\frac{\beta}{2}\right) = \frac{W}{2x} \]

Now, the overall probability of impact from a projectile is given by:

\[ P_{\text{impact}} = P_{xy}P_r \quad \text{(Equation 6)} \]

Solving equations 2, 4, 5 & 6 for probability of impact on a given target size and a range of initial velocities gives interesting results when plotted against distance. In Figure 3 are the probability results for a target the approximate size of a human, 1.83m by 0.6m, for initial projectile velocities of 10m/s through to 300m/s. What is interesting is that the probability curves lie on the same line for the majority of the distance to the target, only deviating as the maximum range of the projectile is approached, \( x_{\text{max}} = \frac{V^2}{g} \), with no drag. This increase in probability as \( x_{\text{max}} \) is approached is due to the increased contribution of the high trajectory to the overall probability. Up to this point the probability of impact is dominated by the low, most direct, trajectory to the target. The contribution of each of the trajectories can be seen in Figure 4 for a single projectile with initial velocity of 200m/s. The steep angle of attack on the target from a projectile following a high trajectory reduces the contribution to the target. The contribution of each of the trajectories can be seen in Figure 4 for a single projectile with initial velocity of 200m/s. The steep angle of attack on the target from a projectile following a high trajectory reduces the contribution to the overall probability as the target appears smaller to the on-coming projectile. As the low trajectory and high trajectory approach \( x_{\text{max}} \), the contribution to overall probability tends to the same value.

The apparent independence of probability of impact on the initial velocity leads to the question: is there a simpler method that would produce this result?

### A Simplified Approach

As the contribution from the high trajectory is small, consider only the low trajectory, and for that just consider a straight line to simplify the geometry. Also, for the radial probability consider the width of the target to be small compared to the radius of the circle of possible targets, so the trigonometric functions can be dropped, see Figure 5. With these assumptions, the probability of the \( x-\)y plane and for the radial distribution are given by:

\[ P_{xy} \sim \frac{a}{\pi} = \tan^{-1}\left(\frac{H}{x}\right) \frac{1}{\pi} \quad \text{for small} \ H/x \]

And

\[ P_r \sim \frac{W}{2\pi x} \]

For a completely random distribution of projectile launch directions, only half of the projectiles will travel in the direction of the target. If the initial elevation is right at ground level then only half of these projectiles will travel in an upward direction towards the target. Therefore, the upper limit for \( P_{\text{impact}} \) which will be called \( P_{\text{max}} \) is 0.25 if \( y_{\text{mc}} = 0 \) and 0.5 if \( y_{\text{mc}} > 0 \). Now, the probability of impact from the simplified analysis is given by:

\[ P_{\text{impact}} = \min\{P_{\text{max}}, P_{xy} \times P_r\} = \min\left\{P_{\text{max}}\left(\frac{WH}{2\pi x}\right)^\frac{1}{2}\right\} \quad \text{(Equation 9)} \]

Comparing the simplified analysis in equation 9 with the more rigorous trajectory results show good agreement, as can be seen in Figure 6 (this analysis for ground level release of projectiles). The major deviation is at very short distances, around a meter, where blast wave impact would dominate impact. For the remainder of the curve the simplified analysis and trajectory analysis superimposed, ignoring the “up-tick” near \( x_{\text{max}} \).
Figure 3: Probability of impact from a single projection as a function of distance for a range of initial projectile velocities

Figure 4: Contribution to the overall probability of impact from the low trajectory and the high trajectory for an initial velocity of 200m/s
The Effect of Atmospheric Drag

As the fragments involved in the failure of a pressurised system could range in size and shape, the effect of drag has also been considered. A trajectory analysis was used, and the governing equations are a little more involved:

\[
\frac{d^2y}{dt^2} = -g - A_p \cdot C_d \cdot \rho_{air} \cdot \frac{V_y^2}{2m_p} \tag{10}
\]

\[
\frac{d^2x}{dt^2} = -A_p \cdot C_d \cdot \rho_{air} \cdot \frac{V_x^2}{2m_p} \tag{11}
\]

Equations 10 and 11 are not amenable to analytical solution, so numerical integration and a statistical study of the parameters was used to determine the trajectories that recorded a hit on the target. The statistical study performed individual trajectory
calculations with drag for a large number of projectiles, each of which has the same release velocity and an equal chance of launching in any direction. Firstly, this analysis was done with no drag to compare the results with the analytical method. An example of the comparison between the two solution methods can be seen in Figure 7, with very good agreement, and the numerical results fall on the simplified analysis line also. Results are for ground level release of projectiles that are small in size compared to the target.

Figure 7: Comparison of Analytical results with Numerical results

Figure 7: Comparison of Analytical results with Numerical results

Projectile drag coefficients can vary significantly based on shape. One would expect there to be large flat projectiles produced in a pressurised failure, with a resulting large drag coefficient. Calculations were done for large projectiles with a range of drag coefficients, from 0.5 to 2. It is assumed that the projectiles only have drag and no lift, which will be true for most “chuck” shaped randomly spinning projectiles. Even with the drag coefficient accounted for, the results still fall on the same basic curve, as seen in Figure 8. The same increase near $X_{\text{max}}$ is seen in the numerical simulation as in the analytical solution, for the same reasons.

Figure 8: Numerical results for large projectiles with a range of drag coefficients

As a result of these three approaches we conclude that one simple curve can be used to predict the probability of impact on any vertical target by a single projectile, up to a distance $x \sim X_{\text{max}}$, the only variable being the size of the target.

Target Area, allowing for the size of projectile

The analysis up to this point has been based on projectiles that are small compared to the area of the target. Large projectiles do not have to hit the target full-on, if the target is hit by any part of the projectile, the effect will be assumed to be the same. The implication of this is that the effective target area is increased by the size of the projectile, as seen in Figure 9. The target can be hit from above, either side, and below – to allow for the projectile skidding along the ground. So, the effective area of the target is given by:

$$A_{\text{target}} = (H + D_p) \times (W + D_p)$$ \hfill (12)

The projectile equivalent average diameter is given by:

$$D_p = \sqrt{\frac{4A_{\text{vessel}}}{n}}$$ \hfill (13)

Where $n$ is the number of fragment the vessel breaks into. So, equation 9 becomes:

$$P_{\text{impact}} = \min \left\{ P_{\text{max}} \left( \frac{A_{\text{target}}}{2\pi x^2} \right) \right\}$$ \hfill (14)
Number of Projectiles

The number of projectiles produced by a vessel failure depends upon the nature of the failure (Baker 1983, CCPS 2010). A conservative value for the number of particles should allow for additional risk associated with secondary projectiles not specifically included in the analysis. Secondary projectiles are items that are picked up in the blast wave and accelerated along a hazardous trajectory.

A number of failure modes can be considered and a suggested number of fragments is proposed based on Air Products experience (Figure 10Figure 11Figure 12) and the cited reference (Baker 1983):

- Failure near operating / design pressure: due, say, to an external fire. Typically, 2 to 10 projectiles would be expected – use \( n = 10 \). See example in Figure 10
- Failure near ultimate pressure: due to overpressure, internal deflagration etc. Typically, 30 to 100 projectiles would be expected – use \( n = 100 \). See example in Figure 11
- Brittle failure: due to internal detonation, cooling below ductile–brittle temperature, with overpressure, etc. Typically, >100 fragments, used \( n = 500 \). See example in Figure 12

When there are multiple projectiles, the probability of being struck increases since the projectiles are moving in multiple directions which are assumed to be random. As the target can be hit by one projectile or another, the common OR logic is used to calculate the cumulative probability of the target being hit by one projectile:

\[
P_{\text{impact}} = 1 - (1 - P_{\text{impact}})^n
\]

- 15

Impact Consequence

Outdoor vulnerability of personnel to projectiles has been studied and data collated in a number of ways, an example of such vulnerability curve from Baker 1983 can be seen in Figure 13. A conservative estimate would be to assume that there is the same vulnerability for people inside a typical industrial building as there is outside. Failure of process pressure vessels will usually generate projectiles of sufficient mass and velocity to cause a high probability of fatality, so it is assumed if personnel are hit a fatality occurs:

\[
P_{\text{vulnerability}} = 1
\]

- 16

So, the probability of fatality from projectiles is given by:

\[
P_{\text{fatality}} = P_{\text{impact}} \times P_{\text{vulnerability}}
\]

- 17
Effect of Elevation on Probability of Impact

The initial elevation of the projectiles, say from an elevated pipe bridge or vessel within an open structure, will change the angle of attack for targets close to the incident, as well as the overall distance a projectile can travel. The initial elevation of the projectiles was included in equation 4 as \( y_0 \), and in Figure 3, Figure 4 and Figure 6 the elevation is set to zero. To include cases where the initial elevation is greater than zero, a good fit between the simplified analysis and the rigorous analysis can be obtained through a simply modification of equation 14, see Figure 14, which when combined with equations 15, 16 and 17 provides the general solution, equation 18.

\[
P_{\text{fatality}} = 1 \times \left(1 - \min \{ 1 - \frac{A_{\text{target}}}{2\pi(y_0^2 + x^2)} \} \right) - 18
\]

Comparing the results from equation 18 with those from equation 4, including different elevated launch positions, shows the simplified analysis is a good approximation to the rigorous analysis, see Error! Reference source not found. (analysis is for small projectiles). There is very good agreement, apart from at very short distances where the \( \tan(\alpha) \) approximation is not valid, however, these short ranges are of little practical interest.

Figure 13: Example of vulnerability data based on projectile weight and impact velocity.

Figure 14: Simplified method to include elevated
Figure 15: Comparison of simplified and rigorous models for elevated launch position

**Initial Projectile Velocity and Maximum Distance**

The initial velocity of a projectile and the launch angle will determine how far the projectile will go. Clearly any target beyond the maximum distance will not be hit. This analysis has not considered either rocketing of the projectiles, such as a compressed gas cylinder, or bouncing of projectiles across the ground after the initial impact.

The probability of fatality for a target beyond maximum range of the projectile is zero. So, to determine the maximum range of a projectile from the conditions of the failing vessel or pipe, the analysis presented in CCPS 2010 is used:

$$x_{\text{max}} = \frac{R m_p}{\rho_{\text{air}} C_d A_p}$$  \hspace{1cm} (19)

$\bar{R}$ is the scaled distance taken from a correlation developed from data in the same reference as equation 19:

$$\bar{R} = \exp \left\{ 0.0003 \ln(\bar{V})^4 - 0.0018 \ln(\bar{V})^3 - 0.061 \ln(\bar{V})^2 + 0.7255 \ln(\bar{V}) - 0.3242 \right\}$$  \hspace{1cm} (20)

Where:  

$$\bar{V} = \frac{\rho_{\text{air}} C_d A_p V^2}{m_p g}$$  \hspace{1cm} (21)

$$E_k = \frac{k P V}{(y-1)}$$

$$k = \left( 1 - P_R \right)^{y-1/y} + (y-1) P_R \left( 1 - P_R^{-1/y} \right) / 2$$

$$P_R = \frac{P_{\text{atm}}}{P}$$

Using equations 19, 20 and 21 the pressure at failure and the dimensions of the vessel give a maximum range for the projectiles, and so a limit on the range at which a fatality can be expected.

**Comparison with Blast Wave Vulnerability**

Failure of a pressure vessel will produce a blast wave as well as possible projectiles. There are well established methods for assessing the impact of a blast wave on building and personnel (Baker 1983, Oswald 2000) based on over-pressure and impulse. A comparison of the probability of fatality from blast wave and projectiles following a pressure vessel failure at 100barg is given in Figure 16. The vessel size is 10m long by 2m diameter, and curves are given for a range of projectile number, reflecting a number of different failure scenarios. The over-pressure and impulse from the vessel failure were calculated using the Baker PV blast model, (Geng 2011). Probability of fatality due to blast over-pressure and projectiles were determined for personnel both in the open and within a building (steel frame with metal cladding).
Figure 16: A comparison of the probability of fatality from blast wave and projectiles following a pressure vessel failure at 100 barg

In Figure 16, it can be seen that blast vulnerability for people outside dissipates relatively quickly, whereas vulnerability for people inside buildings extends further, due to potential for building damage or collapse resulting fatality. It should be noted that the vulnerability of outside personnel can be higher from projectiles than from blast overpressure. Prior to this assessment methodology, this vulnerability would typically not have been included in risk assessments.

Conclusions

A new simple model has been developed for predicting probability of a projectile resulting from vessel failure hitting a vertical target. The model compares well to more rigorous analytical models and numerical-statistical models which include projectile dimensions and drag. The simple model has been extended to include multiple projectiles and to predict the overall probability of fatality.

For typical industrial process equipment, results are robust and independent of nearly all parameters usually of concern when modelling projectile trajectories, such as distribution of projectile masses, velocities, drag coefficients, etc. Results are also relatively insensitive to the assumed number of projectiles.

Assumptions in the model development are limited:

- Relatively even distribution of projectile trajectories in all directions. If it is expected that projectiles will only travel in certain directions, method could be modified relatively easily to accommodate preferential trajectories.
- An estimate of number of projectiles is based on knowledge of vessel failure type. If desired, sensitivity to number of projectiles can be performed as part of the risk assessment.
- Average projectile size is based on the size of the vessel and the assumed number of projectiles

When compared to risk from blast waves, including impact of projectiles significantly increases vulnerability for people outside, but has more modest impact on risk for people inside buildings.

This simple model lends itself well to being used in a spreadsheet tool which can then be used with other methods to give a fuller assessment of risk. Further validation of this approach is required and this technique cannot be relied up-on in isolation of other assessment techniques and scenario evaluation. Those performing a risk assessment of any given hazardous scenario are responsible for validation of specific hazards and risk estimates used in making management decisions related to personnel safety.
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Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
<th>Unit of Measure</th>
<th>Symbol</th>
<th>Definition</th>
<th>Unit of Measure</th>
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<tr>
<td>( A_p )</td>
<td>Cross-sectional area of projectile</td>
<td>m²</td>
<td>( \bar{R} )</td>
<td>dimensionless distance</td>
<td>-</td>
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<tr>
<td>( A_{\text{target}} )</td>
<td>Area of the target</td>
<td>m²</td>
<td>( t )</td>
<td>time</td>
<td>s</td>
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<td>( C_d )</td>
<td>Drag coefficient</td>
<td>-</td>
<td>( V )</td>
<td>Volume of vessel/pipe from which projectiles come</td>
<td>m³</td>
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<td>( D_p )</td>
<td>equivalent diameter of projectile</td>
<td>-</td>
<td>( \bar{V} )</td>
<td>dimensionless velocity - equation 21</td>
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<td>( E_k )</td>
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<td>( V_o )</td>
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<td>( \beta )</td>
<td>Radial trajectory angle</td>
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<td>Ratio of specific heat ( C_p/C_v )</td>
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