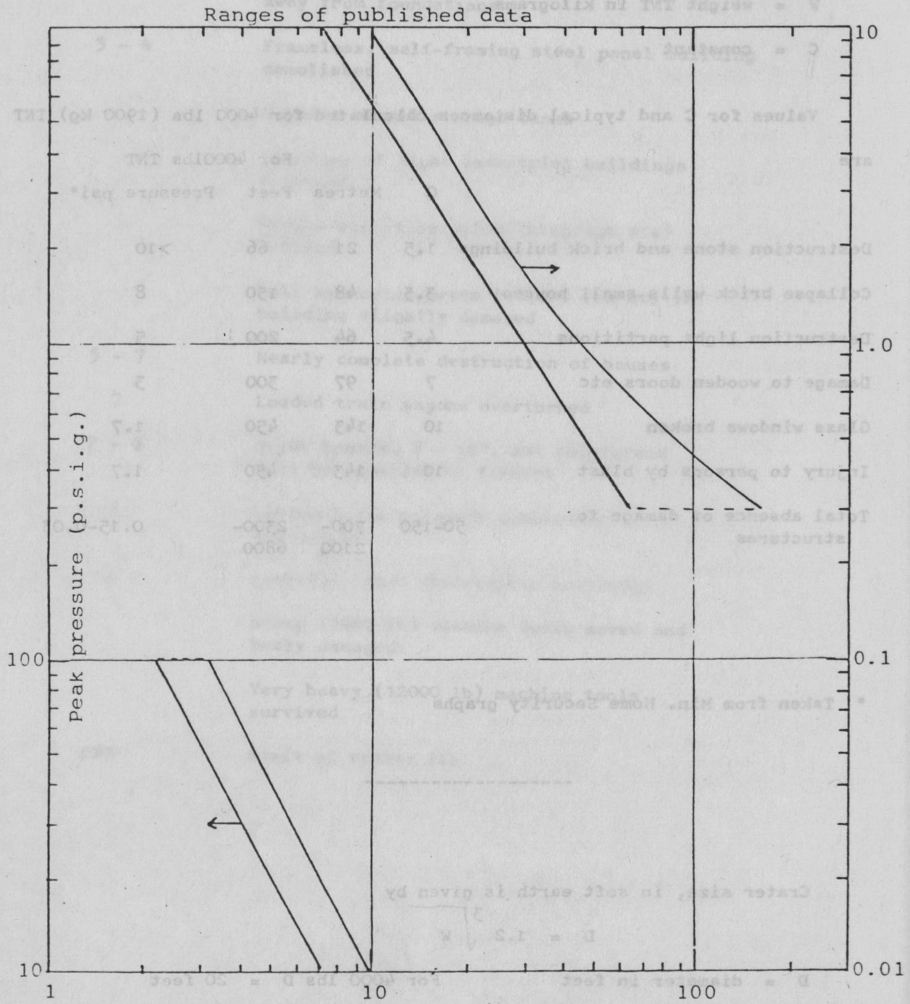
Appendix 2. Peak side-on over-pressure v. distance for TNT.



Scaled distance (ft/lbs$^{1/3}$)
(Values unreliable beyond broken lines)

THE THRUST ON THE SUPPORTS OF A TWO CHAMBER VESSEL WHEN THE BURSTING DISC IN THE DUCT CONNECTING THE CHAMBERS IS RUPTURED

W H L PORTER

A method is presented for estimating the maximum thrust on the supports of a reactor vessel connected by way of a duct to a container vessel when the bursting disc in the duct is ruptured. The method is based on an examination of the acceleration of the centre of gravity of the contents of the vessels.

INTRODUCTION

Many hazardous reactions on the chemical industry are performed in autoclave reactor vessels which can be relieved from the effects of overpressure by a bursting disc. Since the contents of the reactor may be toxic or inflammable it is often connected by a duct containing the bursting disc to a container vessel, which accepts the release without allowing it to escape to the environment. When the bursting disc first fails a force will be transmitted to the restraining structure supporting the vessels and it is the purpose of this report to examine the behaviour of the contained gases when rupture of the bursting disc first occurs and the size of the resulting thrust on the structure. The investigation was stimulated by the work of Dr W A Woods and his co-workers. (Ref 4 and 5).

THE BEHAVIOUR OF THE CONTAINED CASES

This paper develops the argument for an ideal gas which can however have an entirely different composition either side of the bursting disc. When the disc first ruptures a shock wave travels downstream towards the container vessel from the bursting disc. This shock wave becomes steeper and sharper as it progresses. At the same time the high pressure end of a rarefaction wave travels upstream towards the reactor vessel, this rarefaction wave becomes ever more extended as it progresses and indeed with higher initial pressure ratios between the two vessels the low pressure end of the rarefaction wave moves in the opposite direction to the high pressure end; in other words, the wave elongates in such a way that its high pressure end is moving towards the reactor vessel and its low pressure end towards the container vessel.

This paper argues that since there is no external thrust from ejected material from the total system, the thrust on the supports must equal the mass of enclosed material times the acceleration of its centre of gravity (Newtons Law), or if the system is entirely unsupported in free frictionless space the vessels will move in such a way that the centre of gravity remains motionless.

The interface between the two gases, which is determined initially by the position of the diaphragm, moves towards the container vessel immediately after the diaphragm is ruptured. The pressure and velocity, but not the temperature, are considered to be identical on either side of the contact surface and the situation is idealised by assuming that no mixing occurs across the surface.

It follows that:

$$P_2 = P_3 \qquad (1)$$

and also since the velocity of gas is identical on either side of the contact surface

$$U_2 = U_3$$

Now the relationships for the velocity of a gas behind a shock front and behind an expansion front are well-known and are given in references 1 and 2.

$$U_2 = a_1\left(\frac{P_2}{P_1} - 1\right)\sqrt{\frac{2/\gamma_1}{(\gamma_1 + 1)\frac{P_2}{P_1} + (\gamma_1 - 1)}} \qquad (2)$$

and

$$U_3 = \frac{2a_4}{\gamma_4 - 1}\left[1 - \left(\frac{P_3}{P_4}\right)^{\frac{\gamma_4 - 1}{2\gamma_4}}\right] \qquad (3)$$

From which it follows that:-

$$\frac{P_4}{P_1} = \frac{P_2}{P_1}\left[1 - \frac{(\gamma_4 - 1)\,(a_1/a_4)\,(\frac{P_2}{P_1} - 1)}{\sqrt{4\gamma_1^{\,2} + 2\gamma_1(\gamma_1 + 1)(\frac{P_2}{P_1} - 1)}}\right]^{-\frac{2\gamma_4}{\gamma_4 - 1}} \qquad (4)$$

where

$$a_1 = \sqrt{\frac{\gamma_1 P_1}{\rho_1}} \quad : \quad a_4 = \sqrt{\frac{\gamma_4 P_4}{\rho_4}} \qquad (5)$$

Also it can be shown:

$$\frac{T_3}{T_4} = \left(\frac{P_1}{P_4}\right)^{\frac{\gamma_4 - 1}{\gamma_4}} = \left[\frac{P_2/P_1}{P_4/P_1}\right]^{\frac{\gamma_4 - 1}{\gamma_4}} \qquad (6)$$

Note $P_2 = P_3$

Also

$$\frac{T_2}{T_1} = \frac{(\gamma - 1)P_2 + (\gamma + 1)P_1}{(\gamma + 1)P_2 + (\gamma + 1)P_1} \qquad (7)$$

For the shock front

$$U_s = a_1\sqrt{\frac{1}{2\gamma_1}\left[(\gamma_1 - 1) + (\gamma_1 + 1)\frac{P_2}{P_1}\right]} \qquad (8)$$

And for the expansion wave

$$U_r^+ = -a_4 \qquad (9)$$

$$U_r^- = -a_4 + \frac{\gamma + 1}{2}U_p \qquad (10)$$

where $U_p = U_2 = U_3$

Also

$$\frac{\rho_3}{\rho_4} = \left(\frac{P_3}{P_4}\right)^{\frac{1}{\gamma_4}} \qquad (11)$$

and

$$\frac{\rho_2}{\rho_1} = \frac{(\gamma + 1)P_2 + (\gamma - 1)P_1}{(\gamma + 1)P_1 + (\gamma - 1)P_2} \qquad (12)$$

It is to be noted that the velocities, densities and pressures of each zone are constant in time.

Now the acceleration of the centre of gravity of the gas will be considered:

After time t the situation is as in Figure 1 where:

$$d_1 = L_2 \; : \; d_2 = U_r^+ t \; : \; d_3 = (U_r^- - U_r^+)t \; : \; d_4 = (U_p - U_r^-)t$$

$$d_5 = (U_s - U_p)t \quad \& \quad d_6 = (L_1 - U_s t) \qquad (13)$$

Also assuming the density in the rarefaction wave changes linearly with distance along its length.

$$d_3' = \frac{d_3}{3}\left[\frac{\rho_3 + 2\rho_4}{\rho_3 + \rho_4}\right] \qquad (14)$$

Taking movements for the gas about 0-0

$$-\rho_1 V_1 X_1 + \rho_1 A_t \frac{d_6^2}{2} + \rho_2 A_t d_5 \left[d_6 + \frac{d_5}{2}\right] + \rho_3 A_t d_4 \quad d_6 + d_5 + \frac{d_4}{2} \quad + \ldots$$

$$\ldots + \frac{\rho_3 + \rho_4}{2} A_t d_3 \left[d_6 + d_5 + d_4 + \frac{d_3}{3}\left(\frac{\rho_3 + 2\rho_4}{\rho_3 + \rho_4}\right)\right] + \ldots$$

$$\ldots + \rho_4 A_t \left[d_2 + d_1\right]\left[d_6 + d_5 + d_4 + d_3 + \frac{d_2 + d_1}{2}\right] + \rho_4 V_4 X_4 = MX_g$$

Substituting for d_1 to d_6

$$-\rho_1 V_1 X_1 + \rho_1 A_t \left[\frac{L_1 - U_s t}{2}\right]^2 + \rho_2 A_t (U_s - U_p)t\left[L_1 - \frac{U_s t}{2} - \frac{U_p t}{2}\right] + \ldots$$

$$\ldots + \rho_3 A_t \left[U_p - U_r^-\right] t \left[L_1 - \frac{U_p t}{2} - \frac{U_r^- t}{2}\right] + \ldots$$

$$\ldots + \frac{\rho_3 + \rho_4}{2} A_t \left[U_r^- - U_r^+\right] t \left[L_1 - U_r^- t + \left(\frac{U_r^- - U_r^+}{3}\right) t \left[\frac{\rho_3 + 2\rho_4}{\rho_3 + \rho_4}\right]\right] + \ldots$$

$$\ldots + \rho_4 A_t \left[U_r^+ t + L_2\right]\left[L_1 + \frac{L_2 - U_r^+ t}{2}\right] + \rho_4 V_4 X_4 = MX_g \qquad (15)$$

Differentiating this equation twice with respect to t

$$\rho_1 A_t U_s^{\,2} - \rho_2 A_t \,(U_s^2 - U_p^2) - \rho_3 A_t (U_p^2 - (U_r^-)^2) - \ldots$$

$$\ldots - A_t\left(\frac{U_r^- - U_r^+}{3}\right)\left[U_r^- \,(2\rho_3 + \rho_4) + U_r^+(\rho_3 + 2\rho_4)\right] - \ldots \qquad (16)$$

$$\ldots - \rho_4 A_t (U_r^+)^2 = M\ddot{X}_g$$

Now force F_A on support = $M\ddot{X}_g$ which is constant (17)

Now consider an element of gas of mass m whose boundaries rest at each end of the tube in undisturbed gas and let the centre of gravity of this gas be at a distance y from 0-0.

Then taking moments for this gas and differentiating twice we find that equation (17) is modified as follows:

$$\rho_1 A_t U_s^2 - \rho_2 A_t (U_s^2 - U_p^2) - \rho_3 A_t (U_p^2 - (U_r^-)^2) - \ldots$$

$$\ldots - A_t \left(\frac{U_r^- - U_r^+}{3}\right)\left[U_r^-(2\rho_3 + \rho_4) + U_r^+(\rho_3 + 2\rho_3)\right] - \ldots \qquad (18)$$

$$\ldots - \rho_4 A_t (U_r^+)^2 = m\ddot{y}$$

$$\text{So } M\ddot{X}_g = m\ddot{y} \qquad (19)$$

But considering the forces acting on the mass m

$$(P_4 - P_1) A_t = m\ddot{y} \qquad (20)$$

The above calculations indicate that if the change in position with time of the centre of gravity of the gas is traced by taking moments, and is then integrated twice to obtain the acceleration, then the acceleration is found to be constant, at least until either of the waves meets the end of the duct, assumed to be of constant cross-section area. The acceleration being constant, the thrust on the supports is also constant and is closely approximated by the simple relationship.

$$\text{Thrust on support} = (P_4 - P_1)A \qquad (21)$$

Moreover, since friction and mixing can only serve to reduce the acceleration, this thrust is a maximum.

The thrust from equation (16) where $F_A = M\ddot{X}_g$ and from equation (20) where $F_B = (P_4 - P_1)$ A are given in Table 1, which does not give an exact identity for F_A and F_B because of the linear approximation for density change in the 5th term of equation 15.

If the reactor vents to the atmosphere then from reference 3, the thrust for subcritical conditions is given by:

$$\frac{F_C}{P_4 A_t} = \frac{2\gamma_4}{\gamma_4 - 1}\left(\frac{P_a}{P_4}\right)^{\frac{1}{\gamma_4}}\left[1 - \left(\frac{P_a}{P_4}\right)^{\frac{\gamma_4 - 1}{\gamma_4}}\right] \qquad (22)$$

This equation applies for a pipe with a divergent outlet provided A_t is supplied as the outlet and not the pipe area.

For critical conditions:

$$\frac{F_C}{P_4 A_t} = 2\left(\frac{2}{\gamma_4 + 1}\right)^{\frac{1}{\gamma_4 - 1}} - \frac{P_a}{P_4} \qquad (23)$$

If the flow remains critical and an expansion piece is formed in the release pipe. Then it is of interest to note that the thrust F is given by:

$$\frac{F}{P_4 A_t} = \gamma_4 \sqrt{\frac{2}{\gamma_4 - 1}\left(\frac{2}{\gamma_4 + 1}\right)^{\frac{\gamma_4 - 1}{\gamma_4 - 1}}\left[1 - \left(\frac{P_e}{P_4}\right)^{\frac{\gamma_4 - 1}{\gamma_4}}\right]} + \ldots \qquad (24)$$

$$\ldots + \frac{A_e}{A_t}\left(\frac{P_e}{P_4} - \frac{P_a}{P_4}\right)$$

where P_e is the pressure in the gas at exit and A_e is the exit area and A_t the pipe area. It can be proved that the maximum thrust for a divergent outlet is given when the area of exit is such that the exit gas pressure P_e is identical to the environmental pressure P_a and the thrust is then given by:

$$\frac{F_D}{P_4 A_t} = \gamma_4 \sqrt{\frac{2}{\gamma_4 - 1}\left(\frac{2}{\gamma_4 + 1}\right)^{\frac{\gamma_4 - 1}{\gamma_4 + 1}}\left[1 - \left(\frac{P_a}{P_4}\right)^{\frac{\gamma_4 - 1}{\gamma_4}}\right]} \qquad (25)$$

For critical flow in the pipe the ratio of the pipe area to the outlet area is then given by (Ref 6):-

$$\frac{A_t}{A_e} = \left(\frac{\gamma_4 + 1}{2}\right)^{\frac{1}{\gamma_4 - 1}}\left[\frac{\gamma_4 + 1}{\gamma_4 - 1}\left[\left(\frac{P_a}{P_4}\right)^{\frac{2}{\gamma_4}} - \left(\frac{P_a}{P_4}\right)^{\frac{\gamma_4 + 1}{\gamma_4}}\right]\right]^{\frac{1}{2}} \qquad (26)$$

All four thrusts F_A, F_B, F_C and F_D are given in Table 1 for a range of pressure conditions and also the exit area ratio required to give maximum thrust with a complete break in the connecting duct.

As is to be expected the thrust resulting from a complete rupture of the duct significantly exceeds the thrust from the enclosed system. It also may be noted that the thrust is independent of the volumes of the reactor and container vessels and the total mass content. Moreover not unexpectedly the thrust is proportional to the area of the duct.

CONCLUSION

A method is submitted for calculating the initial thrust on the supports of a totally enclosed system within which two vessels the reactor and the container are connected by a duct and whose contents are separated by a bursting disc. The thrust when the disc first ruptures is significantly lower than for a free jet flow from the duct and its maximum value closely satisfies the simple equation:-

$$\text{Thrust} = (P_4 - P_1)\,A$$

TABLE 1

RESULTS FROM COMPUTER CALCULATIONS

Pressure bars $10^5 N\ m^{-2}$ Density Kg m^{-3}

Case No	P_1	P_2	P_3	P_4	ρ_1	ρ_2	ρ_3	ρ_4
1	1.013	1.013	1.013	1.013	1.177	1.777	1.777	1.777
2	1.013	1.340	1.340	1.793	1.177	1.435	1.691	2.082
3	1.013	1.407	1.407	2.077	1.177	1.496	1.826	2.355
4	1.013	1.883	1.883	3.725	1.177	1.820	2.659	4.331
5	1.013	2.026	2.026	4.398	1.177	1.912	2.936	5.108
6	1.013	2.157	2.157	2.166	1.177	1.994	3.149	5.885
7	1.013	2.491	2.491	7.095	1.177	2.191	3.902	8.240
8	1.013	2.885	2.885	10.135	1.177	2.406	4.799	11.770
9	1.013	3.040	3.040	11.535	1.177	2.484	5.167	13.393
10	1.013	4.101	4.101	25.338	1.177	2.962	8.014	28.425
11	1.013	5.068	5.068	46.553	1.177	3.317	11.089	54.060
12	1.013	7.095	7.095	138.377	1.177	3.894	19.254	160.660

TABLE 1 (Continued)

RESULTS FROM COMPUTER CALCULATIONS

Temperature °K Velocities ms^{-1}

Case No	T_1	T_2	T_3	T_4	U_s	U_p	U_r^-	U_r^+
1	298.0	298.0	298.0	298.0	347.5	0.0	-347.5	-347.5
2	298.0	322.0	274.2	298.0	392.3	70.74	-262.5	-262.5
3	298.0	328.0	269.2	298.0	402.6	85.98	-244.2	-347.5
4	298.0	358.0	245.0	298.0	457.5	161.6	-153.5	-347.5
5	298.0	366.7	238.8	298.0	473.3	182.0	-128.9	-347.5
6	298.0	374.4	233.5	298.0	487.1	199.5	-107.9	-347.5
7	298.0	393.0	221.0	298.0	520.9	241.2	-57.9	-347.5
8	298.0	415.3	208.2	298.0	558.4	285.2	-5.05	-347.5
9	298.0	423.5	203.6	298.0	572.1	301.17	14.06	-347.5
10	298.0	479.3	177.1	298.0	660.2	397.8	130.0	-347.5
11	298.0	528.7	158.1	298.0	730.9	471.5	218.5	-347.5
12	298.0	630.6	127.5	298.0	860.7	600.8	373.4	-347.5

TABLE 1 (Continued)

RESULTS FROM COMPUTER CALCULATIONS

Thrust Newtons

Case No	F_A	F_B	F_C	F_D	Area ratio for maximum thrust AR_D
1	0.0	0.0	0.0	0.0	1.0
2	7210	7236	11660	11660	1.0
3	9363	9416	14460	14460	1.0
4	24740	25237	34510	35270	1.177
5	30670	31447	49390	49390	1.262
6	36550	37663	50260	52170	1.346
7	54130	56490	74150	77770	1.591
8	80060	84730	109900	119400	1.931
9	91850	97720	126400	138400	2.077
10	207600	225900	289030	333200	3.326
11	389600	423000	539000	663600	4.884
12	1270000	1276000	1620000	2038000	9.984

Notes 1) Velocities are +ve in direction from high pressure reactor to low pressure container

2) $a_1 = a_4 = 347.5\ ms^{-1}$ for all cases

3) $MW_1 = MW_4 = 28.2$ (air)

4) $\gamma_1 = \gamma_4 = 1.4$

5) $A_t = 0.0929\ m^2$ ($\equiv$ 1 sq foot)

NOMENCLATURE (SI UNITS)

A_t	Area of connecting duct
A_e	Exit area
AR	Area ratio A_e/A_p (ie area ratio from belling)
a	Sonic velocity
F_A	Thrust on supports given by acceleration of centre of gravity equations 16 and 18
F_B	Thrust on supports given by equation 21
F_C	Maximum thrust resulting from fully breached connecting duct without belling
F_D	Maximum thrust resulting from fully breached connecting duct with belling
g	gravitational constant
L	Linear dimension
m	Mass of gas within undisturbed boundaries in the connecting duct
M	Total mass of enclosed gas
P	Pressure
T	Absolute temperature
t	time
U	Velocity of various fronts in the connecting duct
U_s	Velocity of shock front
U_p	Velocity of contact surface between gases initially on either side of the diaphragm
U_r^-	Velocity of low pressure end of the expansion front
U_r^+	Velocity of high pressure end of the expansion front
V	Volume
X	Linear dimension
γ	Ratio of specific heats
ρ	density

SUFFICES

a	Atmospheric or environmental condition
g	Measured to centre of gravity
t	Throat condition
e	Exit condition of jet
1	Condition in low pressure container vessel 1
2	Condition between shock wave and contact surface
3	Condition between expansion front and contact surface
4	Condition in reactor vessel

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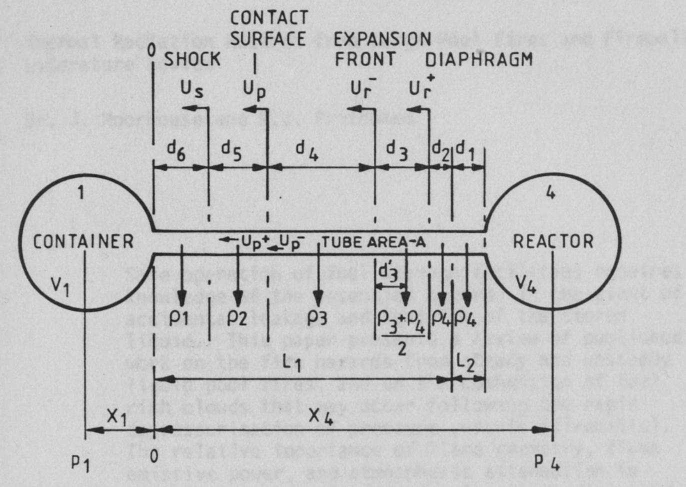

NOTES U_r^- CAN BE POSITIVE OR NEGATIVE IN SIGN
U_r^+ IS ALWAYS NEGATIVE
U_s IS ALWAYS POSITIVE
U_p IS ALWAYS POSITIVE
(POSITIVE INDICATES MOVEMENT FROM RIGHT TO LEFT)

FIG.1. INITIAL TRANSIENT ANALYSIS

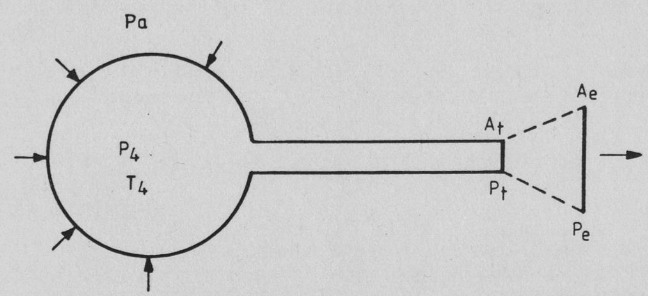

FIG. 2. THRUST ANALYSIS