# A Revised Method for Dropped Object Risk Analysis 

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A revised method for use in dropped object risk analysis is presented. The revised method aims to improve current practice in estimating subsea object excursion and hit probability for dropped object analysis in the context of offshore oil and gas engineering.
A practical problem of some interest is estimation of the probability of an object dropped into the sea, defined by a set of parameters (e.g. density, volume, and shape), arriving at nominated locations on the sea bed (e.g. the set of locations along a pipeline in the vicinity of the drop point).

A first principles approach to characterising the expected dispersion of dropped objects, for cases where the dominant influences on dispersion are unbiased in respect of direction, suggests a classical form of dispersion that is different from that used in current common practice for analysing of such dispersion. This classical form of dispersion provides the basis for a new method of dropped object analysis in general and, in particular for risk assessment of seabed infrastructure.

Relevant published experimental data is shown to conform to the dispersion form at the heart of the new method. A better fit than to the current, widely used, dispersion model is demonstrated.

Application of the new and current "industry standard" method to a model problem is used to highlight the differences of outcome and to discuss the implications for dropped object risk assessment.

The new method predicts a different displacement distribution from the current widely used method. Both methods predict uniform arrival probability at any specified radial displacement. The sum of arrival probabilities should of course be unity for both methods. It follows that the new method predicts a higher arrival probability for some ranges of radius and a lower arrival probability for other ranges of radius. The re-evaluated risk to a seabed object will depend on the details of the case. The value of re-evaluation of a risk using the new method will depend on the sensitivity of the decision which the current method has been used to support. Features of cases that may merit re-evaluation are indicated.

## Introduction

A revised method for use in dropped object risk analysis has been developed and is presented. The revised method aims to improve current practice in estimating object excursion and hit probability for dropped object analysis in the context of offshore oil and gas engineering.

A practical problem of some interest is estimation of the probability of an object dropped into the sea, defined by a set of parameters (e.g. density, volume, and shape), arriving at nominated locations on the sea bed (e.g. the set of locations along a pipeline in the vicinity of the drop point).

## Background

## Dropped Object Study

A frequently applied component of quantitative risk analysis performed for offshore oil and gas installations requires the evaluation of risk, to personnel, to equipment, and to the environment, posed by the potential to drop objects lifted by crane. Lifts may be either between points on the installation or between the installation and an attendant supply vessel. This evaluation is typically carried out as a Dropped Object Study, which may assess the risk of consequences arising from direct impact on personnel, and/or the risk of impact onto topsides equipment, and/or the risk of impact onto subsea pipelines or structures. The latter requires estimation of the possibilities of excursion from the drop point, and so the hit probability, of an object dropped into the sea, and the probable excursion is the key subject of discussion here. Typically, current practice on this topic follows "DNVGL-RP-F107 Risk Assessment of Pipeline Protection" (DNV GL, 2017).

Important variables influencing the path followed by an object dropped into water include (Yasseri 2014):

- The object's shape - e.g. whether long, cylindrical, flat or boxed shaped;
- Mass, drag and inertial coefficients;
- The inclination and orientation of the object at the time of drop;
- The initial velocity at the time of drop;
- The relative position of the centre of buoyancy and the centre of gravity;
- Environmental condition, namely if the sea is calm (sea surface undulation) and the water velocity.

In risk analysis, for shallow water (up to about 100 m depth), a reduced set of variables is used and the effect of orientation and other systematic influences, which are not strongly controlled in practical lifting, are treated as quasi-random contributions to the excursion. In experimental work, many variables may be controlled, so that experimental data typically combines a systematic element and a quasi-random element which should ideally be separated in the interpretation of that data. Variation in the set of influential factors, characterising an object and the circumstances in which it is dropped, leads to a dispersion of possible arrival locations on the sea bed.

## Current Standard Practice

"DNVGL-RP [Recommended Practice]-F107 Risk Assessment of Pipeline Protection" (DNV GL, 2017) is generally considered the "industry standard" methodology for carrying out Dropped Object Studies. This Recommended Practice (RP) provides guidance on the categorisation of lifted objects, the frequency with which lifted objects may be dropped, the distribution of impact location for objects dropped onto the sea, and the damage sustained by subsea pipelines upon impact by a dropped object. The Recommended Practice methodology for determination of impact location distribution is summarised here.

The RP methodology considers an object's deviation horizontally from the point in the horizontal plane above which it was dropped, working in planar polar coordinates with the position vertically below the drop point on the sea bed as the origin. The sea bed is assumed to be a locally flat horizontal plane and any target of interest is assumed to lie in that plane. The methodology uses a normal (Gaussian) distribution

$$
\begin{equation*}
p(x)=\frac{1}{\sqrt{2 \pi} \delta} e^{\left(\frac{-x^{2}}{2 \delta^{2}}\right)} \tag{1}
\end{equation*}
$$

where:
$\boldsymbol{p}(\boldsymbol{x})$ is the probability of a sinking object hitting the sea bottom at a distance $\boldsymbol{x}$ from the vertical line through the drop point; $\boldsymbol{x}$ is the horizontal distance at the sea bottom (metres);
$\boldsymbol{\delta}$ is the lateral deviation (metres).
"Lateral deviation" is a characteristic of the object and is assumed to grow linearly with water depth, $\boldsymbol{d}$, for cases of interest (typical water depths in the North Sea). So, for any case of interest, $\boldsymbol{\delta}$ is calculated from

$$
\begin{equation*}
\delta=d \tan \alpha \tag{2}
\end{equation*}
$$

where $\boldsymbol{\alpha}$ is the angular deviation from the vertical.
"Angular deviation" is a characteristic of the object and is tabulated in the Recommended Practice for shape categories (flat/long shaped, box/round shaped) and mass categories of object.
The methodology describes the probability distribution for an object arriving within a distance $\boldsymbol{r}$ of the vertical line through the drop point in terms of the cumulative distribution function

$$
\begin{equation*}
P(x \leq r)=\int_{-r}^{r} p(x) d x . \tag{3}
\end{equation*}
$$

Whereas, in Equation 1, a "horizontal displacement" was considered without reference to whether $\boldsymbol{x}$ was a Cartesian or polar variable, the analysis that follows from this point implicitly considers radial displacement and performs integration over circles, as in Equation 3, or rings (for incremental probabilities between inner radius $\boldsymbol{r}_{\boldsymbol{i}}$ and outer radius, $\boldsymbol{r}_{\boldsymbol{o}}$ ). However, the integration limits in Equation 3 include negative values of radius (from -r to 0 ) and so are equivalent to using a one-sided distribution from 0 to $r$ with twice the probability density, sometimes described as a 'half-normal' distribution (Tsagris, 2014). Equation 3 is thus equivalent to:

$$
\begin{equation*}
P(x \leq r)=\int_{0}^{r} q(x) d x \tag{4}
\end{equation*}
$$

where

$$
\begin{equation*}
q(x)=p(x)+p(-x)=2 p(x) . \tag{5}
\end{equation*}
$$

In summary, the current Recommended Practice effectively applies a half-normal distribution to the radial probability density.

## Implications of Applying a Half-normal Distribution to the Radial Probability Density

The implications of the application of a half-normal distribution to the radial probability are counterintuitive. The intuitive application of the normal distribution to independent Cartesian axes centred on the point on the sea bed below the drop point is proposed. This distribution is also known as the Rayleigh distribution, in polar form, and has been applied by authors in relevant broadly analogous problems.
The normal distribution as used in the Recommended Practice predicts that a radial probability density (i.e. the probability of hits in very thin rings) will flatten to a maximum as $\boldsymbol{r}$ approaches zero. However, as the circumference (and therefore area) of a thin ring shrinks as $\boldsymbol{r}$ approaches zero, the choice of the normal distribution implies increasing the area density of hits; i.e. the probability of impact per unit area will rise indefinitely (inversely proportional to $\boldsymbol{r}$ ) as $\boldsymbol{r}$ approaches zero. This contrasts with the intuitive sense that area density will reach a maximum and be flat close to $\boldsymbol{r}=\mathbf{0}$ (either for a strong enough dispersion or for a small enough target).

Consider the bullseye - or 'gold' - (a circle of radius $\boldsymbol{r}$ ) and the outer bull - or 'red' - (a ring from $\boldsymbol{r}$ to $2 \boldsymbol{r}$, so having three times the area of the bullseye) on an archery target. A half-normal distribution of radial probability density reaches a finite maximum at $\boldsymbol{r}=\mathbf{0}$. So, for a small enough target or poor enough archer, the radial density at the bullseye and outer bull will be approximately equal. It follows that area density will be three times greater at the bullseye (having one third the area but equal radial density). This suggests that even with a very small target or a very poor archer, the bullseye will be hit as often as
the outer bull (despite the greater area of the outer bull). The intuitively attractive model is that the archer will hit the outer bull more often than the bullseye, and if the archer is poor enough and/or the target small enough, then they will hit the outer bull three times as often (though a good archer would do better with a large enough target).

The intuitive case that area density of hits reaches a finite maximum at $\boldsymbol{r}=\mathbf{0}$ is consistent with a uniform bivariate normal distribution in Cartesian coordinates, which is equivalent to a Rayleigh distribution in polar/radial form, which has been used in broadly analogous cases, e.g. for artillery (Kim, 2002). The predicate for a Rayleigh distribution in this case is that Cartesian displacement (in an arbitrarily selected $\boldsymbol{x}$ direction and a perpendicular $\boldsymbol{y}$ direction in the horizontal plane) is uncorrelated and normally distributed with equal variance and zero mean. The radial probability density in this case is linear in $\boldsymbol{r}$ close to $\boldsymbol{r}=$ $\mathbf{0}$, leading a flat finite maximum in area density at $\boldsymbol{r}=\mathbf{0}$.

No data is presented in the Recommended Practice, or has been found by the present authors, which support choosing a halfnormal distribution. On the contrary, other authors in a broadly analogous case have chosen the Rayleigh distribution (Kim, 2002), a "random walk" analogue suggests applicability of normally-distributed excursion in each direction, and qualitative comparison of the radial probability density exhibited in relevant published data, discussed below, shows a falling density as $\boldsymbol{r}$ approaches 0 (e.g. in Awotahegn, 2015); i.e. consistent with a Rayleigh distribution and inconsistent with a half-normal distribution.

## Proposed Alternative Excursion Probability Distribution

A first principles approach to characterising the expected dispersion of dropped objects, for cases where the dominant influences on dispersion are unbiased in respect of direction, suggests a classical form of dispersion that is different from that used in current common practice for analysis of such dispersion. This classical form of dispersion, the Rayleigh distribution, could provide the basis for a new method of dropped object analysis in general and, in particular for risk assessment of seabed infrastructure.

## Properties of the Rayleigh Distribution

The Rayleigh distribution in radial polar coordinates is an isotropic function of radius $\boldsymbol{r}$ with probability density

$$
\begin{equation*}
f(r)=\frac{r}{\sigma^{2}} e^{\left(\frac{-r^{2}}{2 \sigma^{2}}\right)} \tag{6}
\end{equation*}
$$

where $\boldsymbol{\sigma}$ is the 'scale parameter' which characterises the distribution. The Rayleigh distribution has vanishing probability density at zero radius which grows to a maximum at $\boldsymbol{r}=\boldsymbol{\sigma}$, and then decreases to zero asymptotically as $\boldsymbol{r}$ increases further.

The probability of impact within a radius $\boldsymbol{r}$ of the drop point can be expressed in terms of the cumulative distribution function

$$
\begin{equation*}
F(r)=1-e^{\left(\frac{-r^{2}}{2 \sigma^{2}}\right)} \tag{7}
\end{equation*}
$$

which results in an area density that is approximately constant close to zero.

## Alignment with Recommended Practice

The authors of the Recommended Practice chose a normal distribution and characterised this by angular deviation to fix the "lateral deviation", $\boldsymbol{\delta}$, which is equivalent to the standard deviation. In the RP, an angular deviation is given for each category of object. For a normal distribution, $\sim 68 \%$ of data is expected to lie within one standard deviation of the mean (zero in the present case). For the purpose of discussion in this paper, a corresponding Rayleigh distribution can be matched to the RP lateral deviation by anchoring both distributions to the expectation that $\sim 0.68$ is the cumulative probability of falling within a radius equal to the "lateral deviation", $\boldsymbol{\delta}$ :

The cumulative distribution function of the half-normal distribution (Equation 4) can be evaluated explicitly:

$$
\begin{equation*}
P(x \leq r)=\int_{0}^{r} q(x) d x=\operatorname{erf}\left(\frac{r}{\sqrt{2} \delta}\right) \tag{8}
\end{equation*}
$$

where $\operatorname{erf}(\boldsymbol{r})$ is the error function evaluated at $\boldsymbol{r}$. Taking the case $\boldsymbol{r}=\boldsymbol{\delta}$, Equation 9 confirms the proportion of data (e.g. hits on the seabed) expected to lie within one standard deviation of the origin:

$$
\begin{equation*}
\frac{r}{\delta}=1 \Rightarrow P(x \leq r)=\operatorname{erf}\left(\frac{1}{\sqrt{2}}\right)=0.6827 \tag{9}
\end{equation*}
$$

Rearranging the cumulative distribution function for the Rayleigh distribution (Equation 7) to find the value enveloping the same proportion of data (i.e. where $\boldsymbol{F}(\boldsymbol{r})=\operatorname{erf}\left(\frac{1}{\sqrt{2}}\right)$ ) requires

$$
\begin{equation*}
\frac{r}{\sigma}=\sqrt{2 \times \ln \left(\frac{1}{1-\operatorname{erf}\left(\frac{1}{\sqrt{2}}\right)}\right)} \tag{10}
\end{equation*}
$$

Combining Equation 10 with Equation 9 provides a ratio between the lateral deviation used in the recommend practice and the scale parameter of the Rayleigh distribution which matches at the radius enclosing $\sim 68 \%$ of outcomes in the alternative model presented here:

$$
\begin{equation*}
\frac{\delta}{\sigma}=\sqrt{2 \times \ln \left(\frac{1}{1-\operatorname{erf}\left(\frac{1}{\sqrt{2}}\right)}\right)}=1.5152 . \tag{11}
\end{equation*}
$$

The general form of the Rayleigh distribution conforming to a half-normal distribution according to this ratio is shown in Figure 1, in terms of probability density, and in Figure 2, in terms of the cumulative distribution. The area density of impact probability (the probability per unit area at a radius $\boldsymbol{r}$ from the origin) for each such distribution is shown in Figure 3.


Figure 1 - Probability density functions of the Rayleigh and half-normal distributions, linked by the relationship described in Equation 11.


Figure 2 - Cumulative distribution functions of the Rayleigh and half-normal distributions linked by the relationship described in Equation 11 (i.e. corresponding at cumulative probability ~0.68).


Figure 3 - Area density of impact probability close to the origin for the Rayleigh and half-normal distributions linked by the relationship described in Equation 11.

It can be seen in Figure 3 that the area density of the Rayleigh distribution is approximately constant close to the origin, whereas the area density of the half-normal distribution increases to infinity, producing the counterintuitive results described above.

## Match of the Proposed Excursion Distribution to Experimental Data

Qualitative support for using the Rayleigh form rather than the half-normal form is provided by comparing experimental data with the two distributions, presented in Figure 4. The underlying data presented (Awotahegn, 2015). were chosen for this comparison because they represent an experiment with the fewest directional influences in the selected experimental variables, so that systematic contributions to the experimental excursions are minimised. There is a clear qualitative match of the radial displacement data to the Rayleigh distribution.
The radial data indicate a fall from maximum radial density as $\boldsymbol{r}$ approaches zero, which is consistent with the Rayleigh distribution but not the half-normal distribution (see Figure 1).


Figure 4 -Radial displacement in experimental dropping of model drill pipes with horizontal entry to the water.
Figure 5 shows the expected Cartesian forms of the Rayleigh and half-normal radial distributions in comparison with the $\boldsymbol{x}$ and $\boldsymbol{y}$ displacement results of the same experiments There is a clear qualitative match to the Cartesian equivalent of the

Rayleigh distribution of the separated $\boldsymbol{x}$ and $\boldsymbol{y}$ displacement data (i.e. a match to the bivariate normal distribution underlying the Rayleigh distribution).


Figure 5 - Cartesian displacement in experimental dropping of model drill pipes with horizontal entry to the water.
Further relevant published experimental data are provided by Gilless (Gilless, 2001). Figure 6 shows that the collated data from all experiments reported by Gilless, which included a variety of initial conditions expected to produce various systematic effects, conform qualitatively to the Rayleigh distribution. The qualitative fit of the half-normal at large $\boldsymbol{r}$ appears good. A better fit to the Rayleigh distribution than to the half-normal distribution close to the origin is evident. Close to zero, the experimental data show a clear decrease in frequency from maximum, similar to that predicted by the Rayleigh distribution (this feature is retained and becomes more evident if the data are binned more narrowly). The data presented have been adjusted only by the mean offset in $\boldsymbol{x}$ and $\boldsymbol{y}$ for the whole data set. If instead of using the mean offset from the data to choose the origin, the origin is chosen on the assumption of a Rayleigh distribution, then a better and substantially improved fit to the whole set of data is achieved.


Figure 6 - Comparison of experimental data to the expected (normalised) probability distributions using Rayleigh-and half-normal-distributed excursion, with data binned by radius with a bin width of 0.5 .

If the data are split into sub-categories with different expected systematic effects, then the feature of falling frequency near to $\boldsymbol{r}=\mathbf{0}$ is retained in all cases. Figure 7 shows a clear fit to the Rayleigh distribution of the subset of Gilless experimental data in which the object's centre of mass coincided with its geometric centre, i.e. the subset of data expected to show minimal
systematic error. In this case, a better qualitative fit to the Rayleigh distribution at large $\boldsymbol{r}$ is observed. A better fit to the Rayleigh distribution than to the half-normal distribution close to the origin is again evident.


Figure 7 - Comparison of the set experimental data which are expected to have minimal systematic error to the expected (normalised) probability distributions using Rayleigh- and half-normal-distributed excursion, with data binned by radius with a bin width of 0.5 .

## Application of the Current and Revised Methods to a Model Problem.

Application of the new and current "industry standard" method to a model problem is used to highlight the differences of outcome and to inform discussion of the implications for dropped object risk assessment.

A simple but common problem of real-world interest is the case of a long, straight pipeline passing by an installation. Consider the case of a pipeline of effectively infinite length running parallel to the $\boldsymbol{y}$-axis with its centre crossing the $\boldsymbol{x}$-axis at some $\boldsymbol{x}_{\boldsymbol{p}}$, and an object dropped from above the origin. This is illustrated in Figure 8.


Figure $\mathbf{8}$ - Schematic showing a plan view of the drop point above the origin and a pipeline offset by $\boldsymbol{x}_{\boldsymbol{p}}$ which runs parallel to the $y$-axis.

In the Recommended Practice, the probability of impact is calculated using stepwise summation by considering the area of pipeline contained within concentric annuli. For each annulus, the associated increment of the pipeline area is multiplied by the average probability of impact per unit area (DNV GL, 2017). Far-field annuli (i.e. where $\boldsymbol{r} \gg \boldsymbol{\delta}$ ) contribute negligibly and can be disregarded.

For Rayleigh-distributed object excursion it is simple to calculate the impact probability for the entire pipeline analytically by considering the equivalence of the radial Rayleigh distribution to two Cartesian normal distributions, i.e. by using the bivariate normal form. In the Cartesian form, the Rayleigh distribution has the property of being equivalent to a normal distribution in any section through the horizontal plane, given that a dropped object arrives in that section. The probability of an object of breadth $\boldsymbol{B}$ dropped above $(\mathbf{0}, \mathbf{0})$ impacting a pipeline of diameter $\boldsymbol{D}$ and infinite length centred on $\boldsymbol{x}_{\boldsymbol{p}}$ is therefore

$$
\begin{equation*}
F_{i m p a c t}\left(x_{p}, D, B\right)=\frac{1}{2}\left(\operatorname{erf}\left(\frac{x_{p}+\frac{D+B}{2}}{\sqrt{2} \sigma}\right)-\operatorname{erf}\left(\frac{x_{p}-\frac{D+B}{2}}{\sqrt{2} \sigma}\right)\right) \tag{12}
\end{equation*}
$$

As the problem is symmetrical and the choice of axes is arbitrary, Equation 12 applies to a pipeline of any orientation for which the nearest approach to the origin is $\boldsymbol{x}_{\boldsymbol{p}}$.

For illustration, the impact probability for a pipeline at a range of values for $\boldsymbol{x}_{\boldsymbol{p}}$ has been calculated using both methods. The case considered assumes a point-like object and a pipeline with a diameter of 1 m . The lateral deviation $\boldsymbol{\delta}$ and Rayleigh scale parameter $\boldsymbol{\sigma}$ were calculated based on a water depth of 50 m and an angular deviation of 15 degrees (using the ratio calculated in Equation 11 to fix $\boldsymbol{\sigma})$. The resulting impact probabilities are shown in Figure 9, using an annulus width of 1 m when evaluating the half-normal case using stepwise summation in line with the Recommended Practice, and the ratio of the results from the two methods are shown in Figure 10. The Rayleigh result is analytical so does not require stepwise summation. Figure 11 shows the results for the same case if using an annulus width of 10 m as suggested in the RP, and Figure 12 shows the ratio of results in this case.


Figure 9 - Impact probability for a $1 m$ diameter pipeline of infinite length for different values of nearest approach (offset distance from the drop point) using $1 m$ ring width for the Recommended Practice stepwise summation.


Figure 10 - Ratio of impact probabilities for a $1 m$ diameter pipeline of infinite length for different values of nearest approach (offset distance from the drop point) using $1 m$ ring width for the Recommended Practice stepwise summation.


Figure 11 - Impact probability for a $1 m$ diameter pipeline of infinite length for different values of nearest approach (offset distance from the drop point) using 10 m ring width for the Recommended Practice stepwise summation.


Figure 12 - Ratio of impact probabilities for a $1 m$ diameter pipeline of infinite length for different values of nearest approach (offset distance from the drop point) using 10 m ring width for the Recommended Practice stepwise summation.

It can be seen in Figure 9 that a Rayleigh-distributed excursion produces a curve that is more intuitively appealing, with a maximum if the pipeline passes under the drop point and a smooth decay as the offset distance from the drop point to the pipeline increases.
It appears that the curve predicted for the RP method in Figure 11 broadly approximates that of the Rayleigh-distributed excursion. However, the curve is not smooth and is sensitive to whether the closest approach of the pipeline is close to a multiple of the ring width.

Pipelines of more complex geometry can be approximated as a series of straight segments for the purpose of analysis. To evaluate each segment it is necessary to perform a coordinate transformation to a rotated system $\left(\boldsymbol{x}^{\prime}, \boldsymbol{y}^{\prime}\right)$ where the pipeline segment is oriented such that the pipeline segment runs parallel to the $\boldsymbol{y}^{\prime}$-axis (i.e. $\boldsymbol{x}_{\mathbf{1}}^{\prime}=\boldsymbol{x}_{\mathbf{2}}^{\prime}$ ) with endpoints ( $\boldsymbol{x}_{\mathbf{1}}^{\prime}, \boldsymbol{y}_{\mathbf{1}}^{\prime}$ ) and $\left(\boldsymbol{x}_{1}^{\prime}, \boldsymbol{y}_{2}^{\prime}\right)$ and with $\boldsymbol{y}_{\mathbf{2}}^{\prime}>\boldsymbol{y}_{\mathbf{1}}^{\prime}$.
In the new coordinate system, the probability $\boldsymbol{\Phi}$ of impact by an object of breadth $\boldsymbol{B}$ dropped above $(\mathbf{0}, \mathbf{0})$ onto the pipeline segment is then equal to:

$$
\begin{equation*}
\Phi=\frac{1}{4}\left(\operatorname{erf}\left(\frac{y_{2}^{\prime}}{\sqrt{2} \sigma}\right)-\operatorname{erf}\left(\frac{y_{1}^{\prime}}{\sqrt{2} \sigma}\right)\right) \times\left(\operatorname{erf}\left(\frac{x_{1}^{\prime}+\frac{D+B}{2}}{\sqrt{2} \sigma}\right)-\operatorname{erf}\left(\frac{x_{1}^{\prime}-\frac{D+B}{2}}{\sqrt{2} \sigma}\right)\right) \tag{13}
\end{equation*}
$$

where $\boldsymbol{\sigma}$ is the scale parameter of the object's Rayleigh-distributed excursion.
This allows analytical evaluation of each element contributing to any case of pipelines and similar targets (such as communications cables or umbilicals), whereas the RP uses stepwise approximation and cannot generally be calculated analytically.

## Features of Cases where Re-evaluation should be Considered

The new method predicts a different displacement distribution from the current widely used method. Both methods predict uniform arrival probability at any specified radial displacement from the drop point. The sum of arrival probabilities should of course be unity for both methods. It follows the new method predicts higher arrival probability for some ranges of radius and lower arrival probability for other ranges of radius. The re-evaluated risk to a seabed object will depend on the details of the case.

It has been shown here that if the Rayleigh distribution is the better fit to the data which underlies the RP, and is matched at $\sim 68$ \% inclusion of the data, then it is likely that by following the RP, probability of impact is:

- Overestimated for target elements in the near-field $(\boldsymbol{r}<\mathbf{0} .4 \boldsymbol{\delta}$, from Figure 1);
- Underestimated in the mid-field $(\mathbf{0 . 4 \boldsymbol { \delta }}<\boldsymbol{r}<\mathbf{1} . \mathbf{5} \boldsymbol{\delta})$; and
- Overestimated in the far field $(\mathbf{1 . 5 \boldsymbol { \delta }}<\boldsymbol{r})$.

This tendency to overestimate in the near-field, underestimate in the mid-field, and overestimate in the far-field will be reflected in the outcome of more complex assessments. For example, Table 1 lists the ratios of impact probability predicted for different values of $\boldsymbol{x}_{\boldsymbol{p}}$ for the illustrative case (see Figure 10). The value of $\boldsymbol{\delta}$ used in this case was 13.4 m , and the ratio of results from the two methods is approximately one at $\sim 0.2 \boldsymbol{\delta}$ and $\sim 1.5 \boldsymbol{\delta}$, so these values bound the mid-field in which underestimation of risk by the current RP is indicated.

Table 1 - Illustrative values of impact probabilities predicted from half-normal-distributed excursion and Rayleighdistributed excursion (see also Figure 9, Figure 10)

| Distance $\boldsymbol{x}_{\boldsymbol{p}}$ | Predicted Impact <br> Probability - Half-Normal- <br> Distributed Excursion | Predicted Impact <br> Probability - Rayleigh- <br> Distributed Excursion | Ratio of Half-Normal- <br> Distributed to Rayleigh- <br> Distributed Impact <br> Probability |
| :---: | :---: | :---: | :---: |
| 0 m (near-field) | $8.75 \mathrm{E}-02$ | $4.51 \mathrm{E}-02$ | 1.94 |
| 10 m (mid-field) | $1.72 \mathrm{E}-02$ | $2.38 \mathrm{E}-02$ | 0.72 |
| 30 m (far-field) | $7.87 \mathrm{E}-04$ | $1.44 \mathrm{E}-04$ | 5.48 |

The value of re-evaluating a risk using the new method will depend on the sensitivity of the decision which the current method has been used to support. In particular, the method in the RP overestimates significantly in the far-field compared with the use of the Rayleigh distribution.
For existing assessments it could be worthwhile to reconsider cases where for example, the target of interest lies within $0.4 \boldsymbol{\delta}$ to $1.5 \boldsymbol{\delta}$ or the target of interest is a long, straight pipeline with closest approach within $0.2 \boldsymbol{\delta}$ to $1.5 \boldsymbol{\delta}$, and where the current RP indicated a marginal decision not to mitigate risk, e.g. by protection of the target. For any future assessment, it should be worthwhile applying the proposed new method as a sensitivity check to help inform decision-making.

## Sensitivity to Selection of the Angular Deviation

The current RP provides indicative values for the angular deviation. However, in any real case the caution (or lack of caution) in choosing a particular value of angular deviation is dependent on the range to the target of interest. Generally, in the application of the current RP and where a small single target is of interest, selection of a low angular deviation, such that, lateral deviation $\boldsymbol{\delta}$ is less than the horizontal range to the target will provide a low probability estimate of hitting the target, but also selection of a high angular deviation, such that lateral deviation $\boldsymbol{\delta}$ is greater than range to the target will provide a low probability estimate of hitting the target. For the Rayleigh distribution, the sensitivity to angular deviation is qualitatively similar, but based on the most likely radial displacement $(\sim \boldsymbol{\delta} / 1.5)$. For the current or proposed method, any chosen value of angular deviation will be either more cautious in the near field, and less cautious in the mid and far field, or vice versa.

Sensitivity to the adopted value of angular deviation should be considered. For a cautious assessment, using the current RP, angular deviation could be selected to match the target range if the target is within the cone of the RP recommended angular deviation. Unfortunately, as the data underlying the RP are not publicly available, the confidence that should be attributed to the published values of angular deviation could not be reviewed and any implications for selecting angular deviation if changing to the Rayleigh distribution could not be explored

## Conclusions

A new method for estimating object excursion and hit probability for an object dropped into the sea, by substitution of a Rayleigh distribution for the currently used half-normal distribution, has been proposed as a replacement for this step in current common practice. The Rayleigh distribution is

- More intuitively appealing;
- Used in analogous problems by other authors;
- More consistent with available experimental data;
- Easier to apply (as it can be calculated analytically and so does not require stepwise approximation).

Use of the Rayleigh distribution in replacement of the half-normal (matched at the $\sim 68 \%$ envelope of hits, and with all other variables unchanged) indicates that the current RP possibly overestimates risk in the near field and underestimates risk in the mid field.

Consideration should be given to adopting the Rayleigh distribution to estimate object excursion and hit probability for an object dropped into the sea and for analogous problems. ESR implements the current Recommended Practice in its DROPP software model, for consistency with standard accepted practice, but provides the Rayleigh-distributed excursion basis as an option.

The new method predicts a different displacement distribution from the current widely used method. Both methods predict uniform arrival probability at any specified radial displacement. The sum of arrival probabilities should of course be unity for both methods. It follows the new method predicts higher arrival probability for some ranges of radius and lower arrival probability for other ranges of radius. The re-evaluated risk to a seabed object will depend on the details of the case. The value of re-evaluation of a risk using the new method will depend on the sensitivity of the decision which the current method has been used to support, particularly in the far-field, where the numerical differences are largest. Features of cases that may merit re-evaluation have been indicated.

For any future assessment, it should be worthwhile applying the proposed new method as a sensitivity test to help inform decision making.

Assuming that the current practice was based on fitting to the $\sim 68 \%$ envelope of hits in the underlying data it could be worthwhile to reconsider cases where, for example, the target of interest lies within $0.4 \boldsymbol{\delta}$ to $1.5 \boldsymbol{\delta}$ or the target of interest is a pipeline with closest approach within $0.2 \boldsymbol{\delta}$ to $1.5 \boldsymbol{\delta}$, and where the current Recommended Practice indicated a marginal decision not to mitigate risk.

Separately, the sensitivity to angular deviation in the overall methodology has been discussed. Any chosen value of angular deviation will be either more cautious in the near field or in the mid field.

Sensitivity to the adopted value of angular deviation should be considered. For a cautious assessment, the angular deviation could be selected such that the lateral deviation matches the target range if the target is within the cone of the RP recommended angular deviation, or matches two thirds of the RP lateral deviation where the new method is applied.

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