

CHARACTERIZING IMPELLER PERFORMANCE IN STIRRED TANKS WITH EXAMPLES OF PROCESS RESULTS

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“Mix Thoroughly Until Smooth”
IChemE South Wales Branch

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TOPICS

- Some words on mixing....
- Impeller geometries:
 - Blade shapes etc.
- Re-cap on pump sizing.
- Applying pump rules to impellers in stirred tanks:
 - Defining efficiency.
- Shear characteristics:
 - Time-averaged velocity gradient.
- Dispersion of immiscible liquids.
- Trailing vortex:
 - Look again at dispersion data.
- Conclusions.

IMPORTANCE OF MIXING

- Smith (*Trans IChemE.*, 1990):
 - US chemical industry loses \$ 10×10^9 each year due to poor mixing:
 - 1 % increase in yield $\rightarrow \sim \$ 1 \times 10^6$.
 - One day of down time $\rightarrow \sim \$ 1 \times 10^6$.
- Examples:
 - Low yields in chemical reactions:
 - Change in selectivity on scale-up.
 - By-products, MWD, etc.
 - Longer than expected batch / cycle times.
 - Effluent from WWTP out of compliance.
 - Others.....?

SCALE

- Lab-scale: Vessel diameter 0.1 m (4 inches)
 Operating volume 1 litre (0.25 gall)

- Fine chemicals: Vessel diameter 1.8 m (72 inches)
 Operating volume 8000 litres (2000 gall)

- Pharma fermenter: Vessel diameter 6.0 m (240 inches)
 Operating volume 300000 litres (80000 gall)

- Bio-fuel fermenter: Vessel diameter 18 m (720 inches)
 Operating volume 4 million litres (1 million gall)

- Gasoline storage: Vessel diameter 36 m (1400 inches)
 Operating volume 10 million litres (3 million gall)

6 MILLION GALLON COAL SLURRY TANK

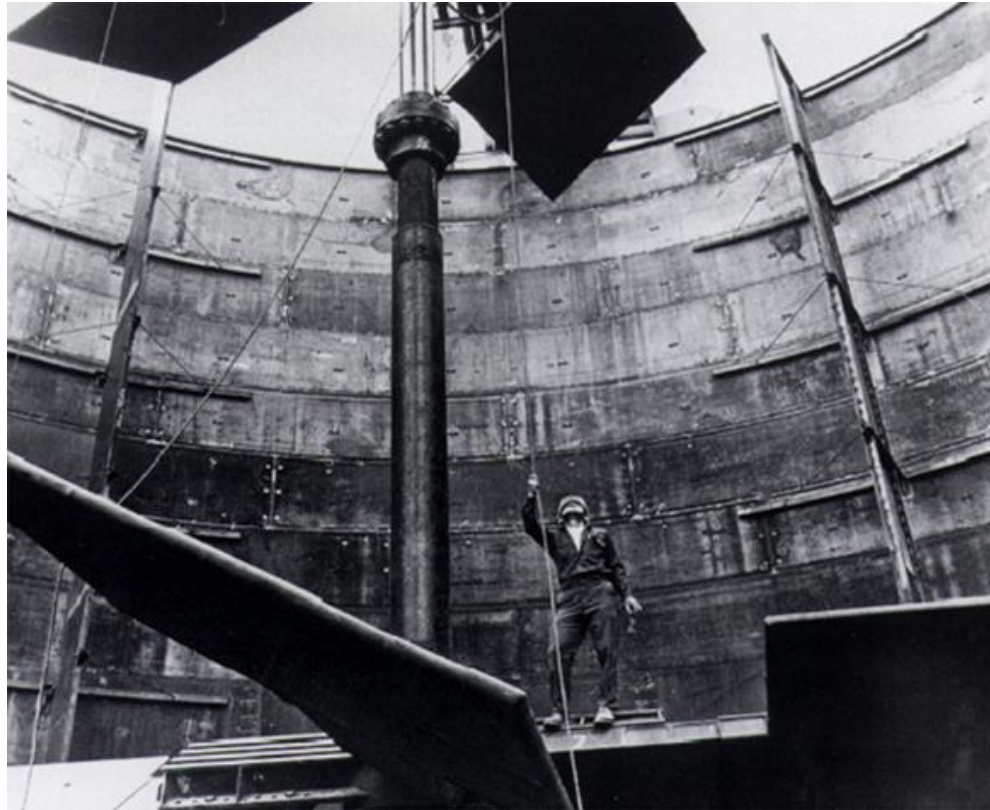
Philadelphia Mixers.

Installed in Mojave Desert.

Coal slurry pumped to Power Generation plant.

Shaft can be raised for start-up.

Then lowered as bed of particles is suspended.



WHAT IS MIXING? (Etchells)

Mixing is the application of

mechanical motion

in order to create

fluid dynamic effects

which achieve a

desired process result.

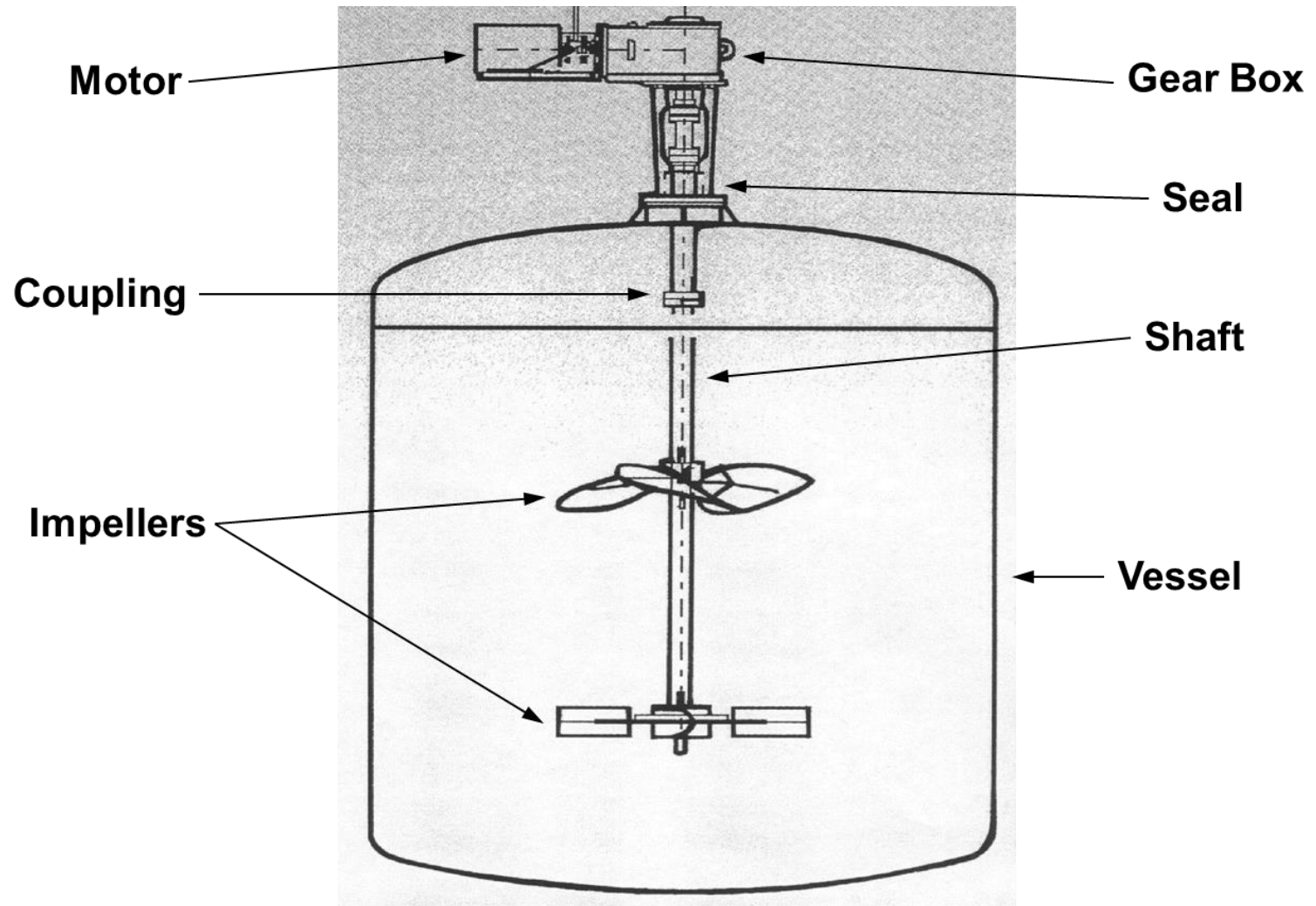
Mean Flow - Velocity

Circulation / Blend Time

Turbulence

Shear

TYPICAL MIXING EQUIPMENT



HYDROFOILS



Low-solidity

$Po \approx 0.30$



Medium-solidity

$Po \approx 0.70$



Anti-ragging

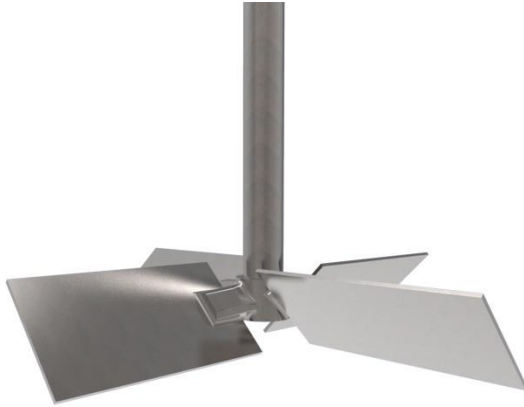
$Po \approx 0.30$

“Low shear” impellers (Ducoste et al, AIChEJ, 1997).

“RAGS”



PITCHED & FLAT BLADE TURBINES



Pitched

$Po \approx 1.30$



Vertical / Flat

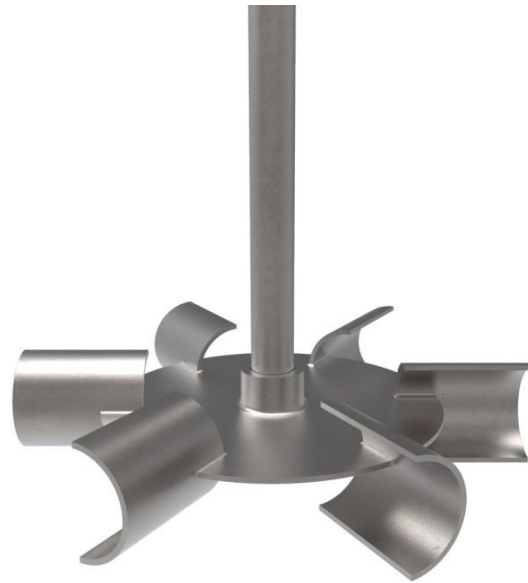
$Po \approx 3.00$

DISC TURBINES



Rushton

$Po \approx 5.00$



Smith

$Po \approx 3.00$



PMSL GDX

$Po \approx 1.00$

“High shear” impellers (Ducoste et al, AIChEJ, 1997).

HIGH-SHEAR / SAWTOOTH



HSD

www.indiamart.com

$Po \approx 0.10 - 0.30$

DEFINING IMPELLER CHARACTERISTICS

- Power number:

$$Po = \frac{P}{\rho N^3 D^5}$$

- Flow number:

$$Fl = \frac{Q}{ND^3}$$

- Diameter ratio:

$$\frac{D}{T}$$

- Projected blade height:

$$\frac{D}{W_p}$$

POWER INPUT TO FLOWING FLUID

- In pipe flow:

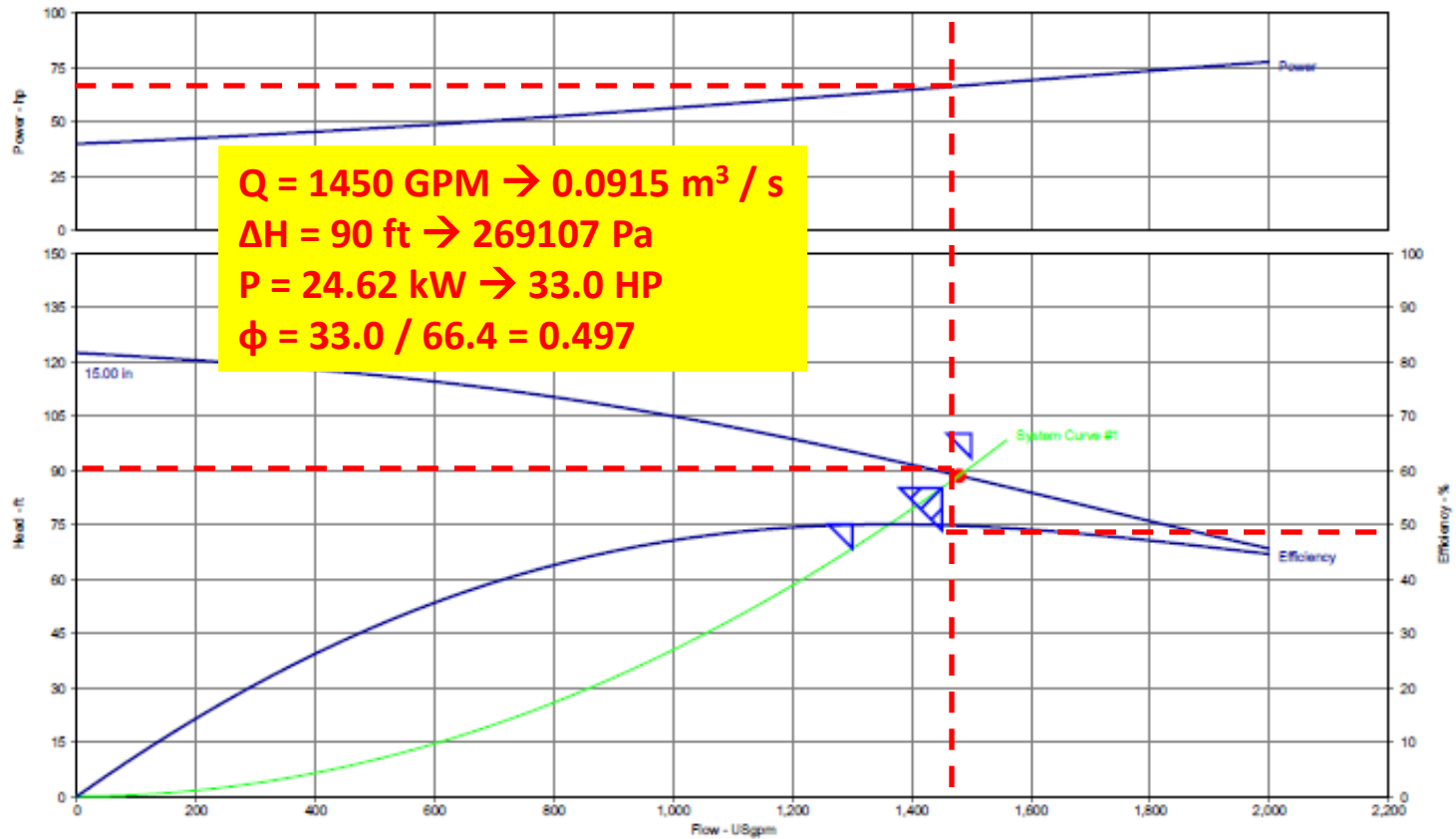
*Note: ΔH is Pressure Drop (Pa) – not Head (m).

$$P = Q\Delta H$$

$$P = \frac{\pi}{4} U D^2 \times 4f \frac{L}{D} \rho \frac{U^2}{2} = \frac{\pi f L}{2 D} \rho U^3 D^2$$

- First term in equation is determined by geometry and flow regime:
 - Reynolds number, pipe roughness.
 - Pipe length to diameter ratio.
 - Fittings (elbows, valves, etc.).

Pump Performance Curve



Customer :	Size : SBG4S75-6T4-15-S	Flow, rated : 1,450.0 USgpm
Customer reference : 31 & 32 Bilge Pumps	Stages : 1	Differential head / pressure, rated : 85.00 ft
Item number : 32 Bilge Pump	Speed, rated : 1200 rpm	Fluid density, rated / max : 1.000 / 1.000 SG
Service :	Based on curve number : SBG4S-6T4-S-15	Viscosity : 1.00 cP
Quantity : 1	Efficiency : 49.77 %	Cq/Ch/Ce/Cn [ANSI/HI 9.6.7-2010] : 1.00 / 1.00 / 1.00 / 1.00
Quote number : 160203CK-1-R0	Power, rated : 66.40 hp	
Date last saved : 09 Feb 2016 12:48 PM	NPSH required : -	

AGITATOR - REYNOLDS NUMBER

- In pipe flow: $Re = \frac{\rho U D}{\mu}$
- What are the appropriate velocity and length scale to use when calculating Reynolds number in a stirred tank?

- Velocity → Impeller tip speed: $U_{TIP} = \pi N D$

- Diameter → Impeller diameter: D

- Drop the π : $Re = \frac{\rho N D^2}{\mu}$

POWER INPUT BY AN IMPELLER

- In pipe flow: $P \propto \rho U^3 D^2$
- Substitute Tip Speed for Velocity (U) as for Reynolds number:

$$P = X\rho N^3 D^5$$

X is the Impeller Power number, P_o . It is determined by geometry and flow regime:

- Reynolds number
- Impeller diameter / Blade width / Number of blades
- Impeller position (relative to vessel base)
- Baffles

TURBULENT & LAMINAR REGIMES

- In turbulent regime:

$$Po = \text{Constant}$$

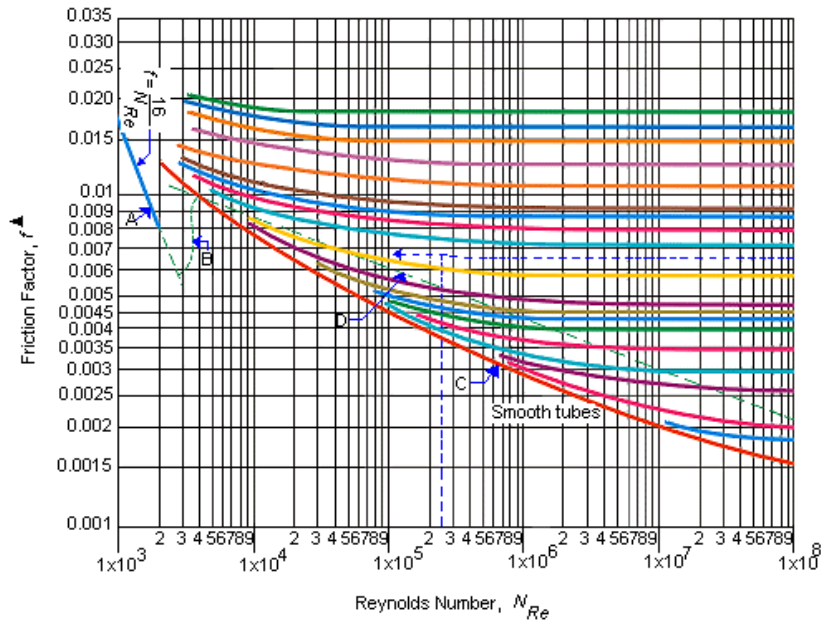
$$P = Po\rho N^3 D^5$$

- In laminar regime:

$$Po = \frac{K_p}{Re}$$

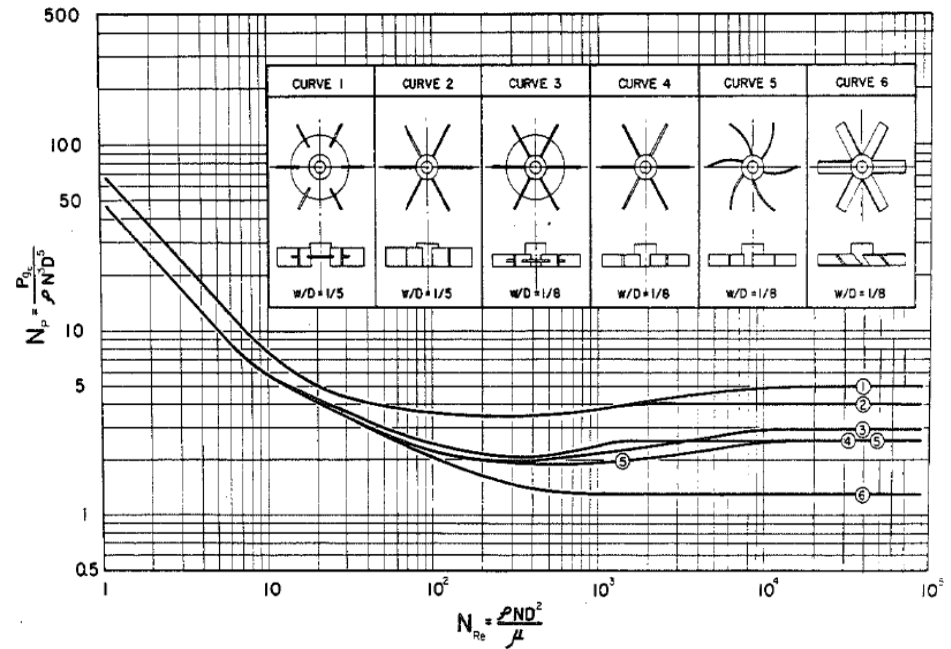
$$P = K_p \mu N^2 D^3$$

FRICTION FACTOR vs. REYNOLDS NUMBER



Turbulent: P_o or $f = k_1$

Laminar: P_o or $f = \frac{k_2}{Re}$



Rushton et al., *Chem. Eng. Prog.*, 1950

POWER, FLOW & EFFICIENCY

- In pipe flow, the Power Input to the Fluid, Flow Rate and Pressure Drop are easy to define and measure.
- If the Motor / Shaft Power is measured, the Pump Efficiency can be quantified.
- How are the Power Input and Flow generated by an impeller measured?

MEASURING POWER

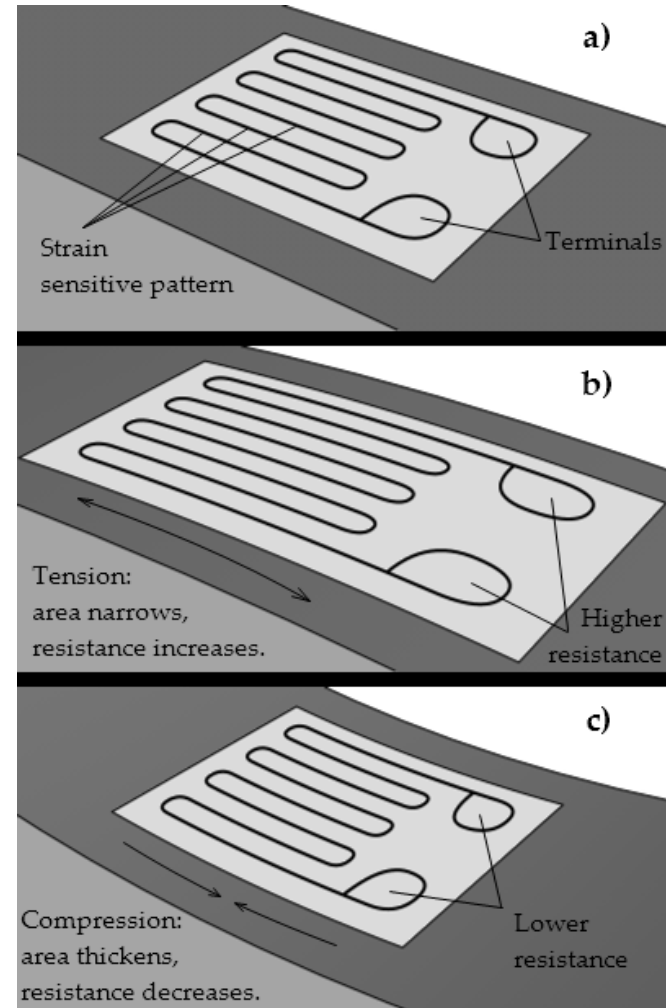
- At lab-scale, use DC motor:

$$P = IV$$

- Use Wattmeter:
 - Direct measurement of power for AC 3-phase motor.
- Measure current:
 - With motor Power vs. Current curve.
- Strain gauges measure torque:

http://en.wikipedia.org/wiki/Strain_gauge

$$Tq = \frac{P}{2\pi N}$$

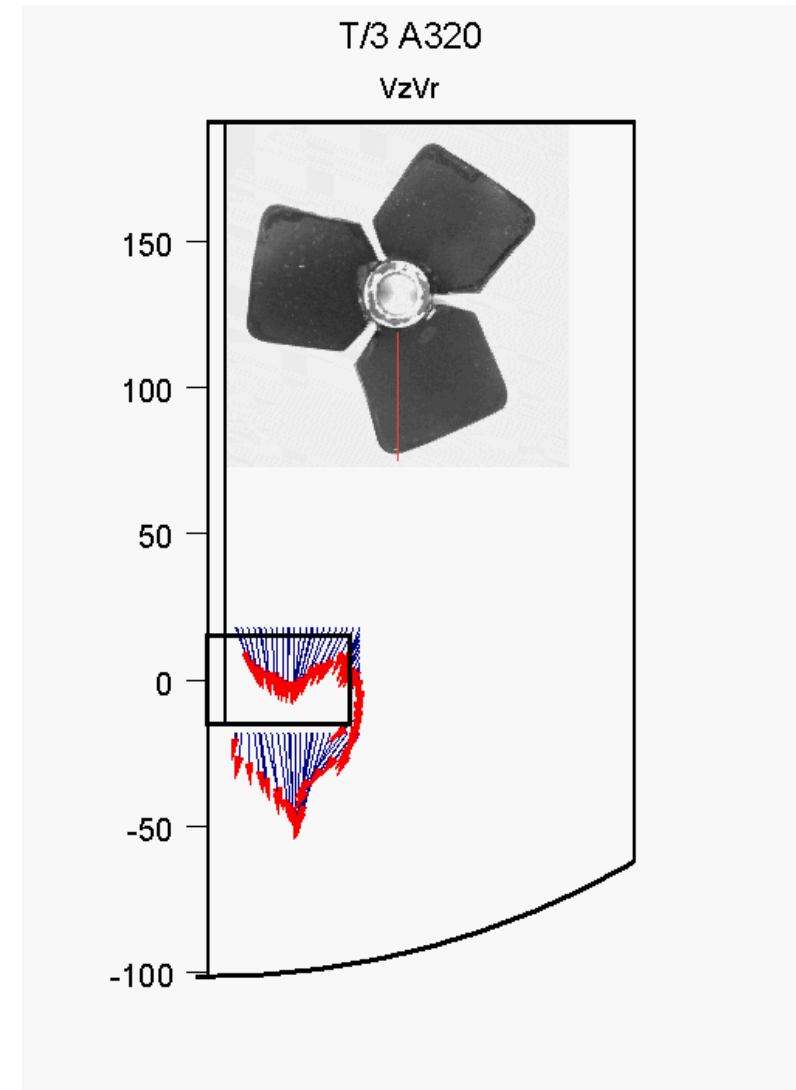


MEASURING FLOW

- Measure velocities in discharge from impeller:
 - Using Laser Doppler Anemometry or Particle Image Velocimetry.
- Flow and velocities are time dependent.
- Calculate mean velocities.
- Define Flow (or Pumping) Number:

$$Q = \iint \bar{U} dA$$

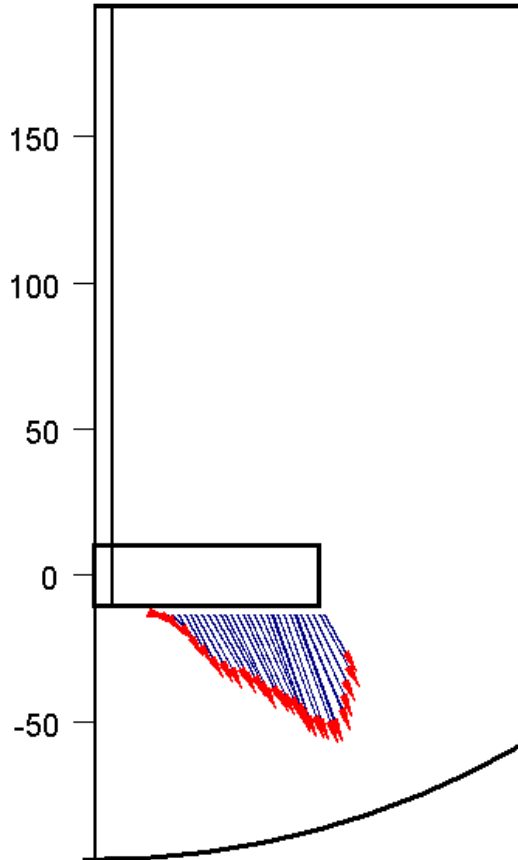
$$Fl = \frac{Q}{ND^3}$$



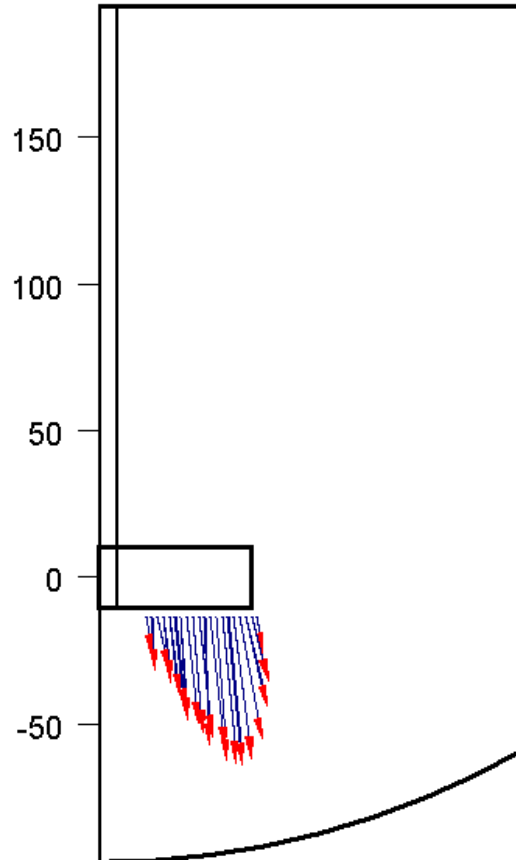
Brown, *MIXING XXII*, 2010

TIME-AVERAGED VELOCITIES

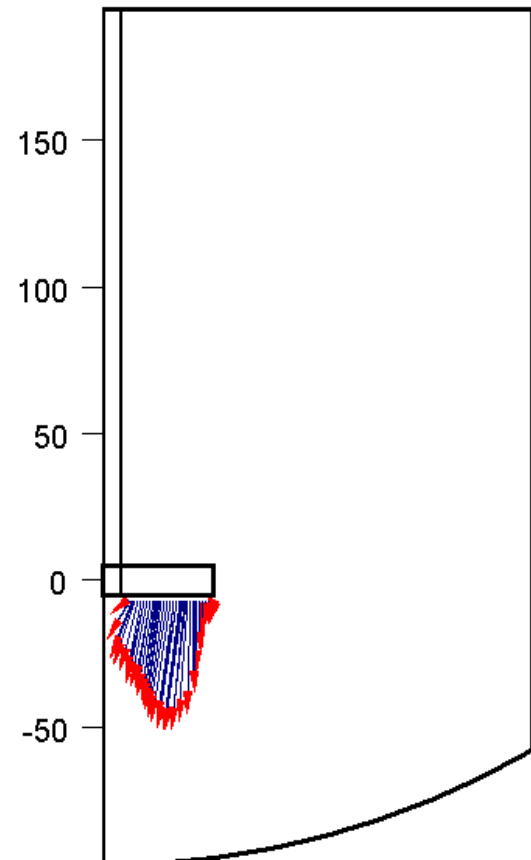
T/2 PBT4-45
C=T/3, Dished base



T/3 PBT4-45
C=T/3, Dished base



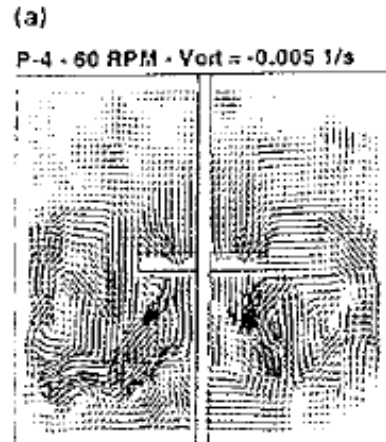
T/4 PBT4-45
C=T/3, Dished base



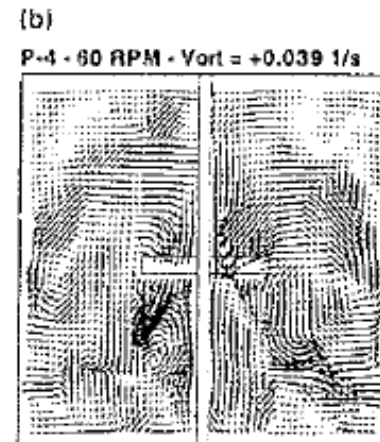
Brown, *MIXING XXII*, 2010

TIME-DEPENDENT VELOCITIES

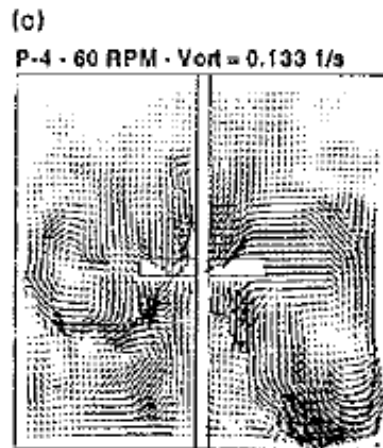
Symmetric flow field
→ low vorticity



Nearly symmetric
flow field → low vorticity



Axisymmetric flow field
→ high +ve vorticity



Axisymmetric flow field
→ high -ve vorticity

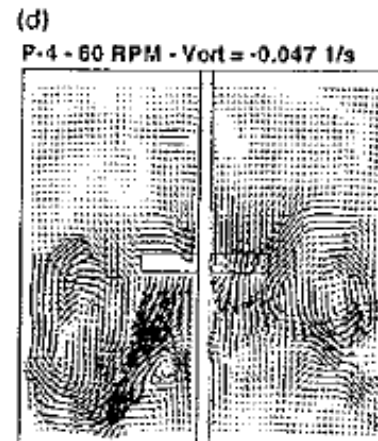
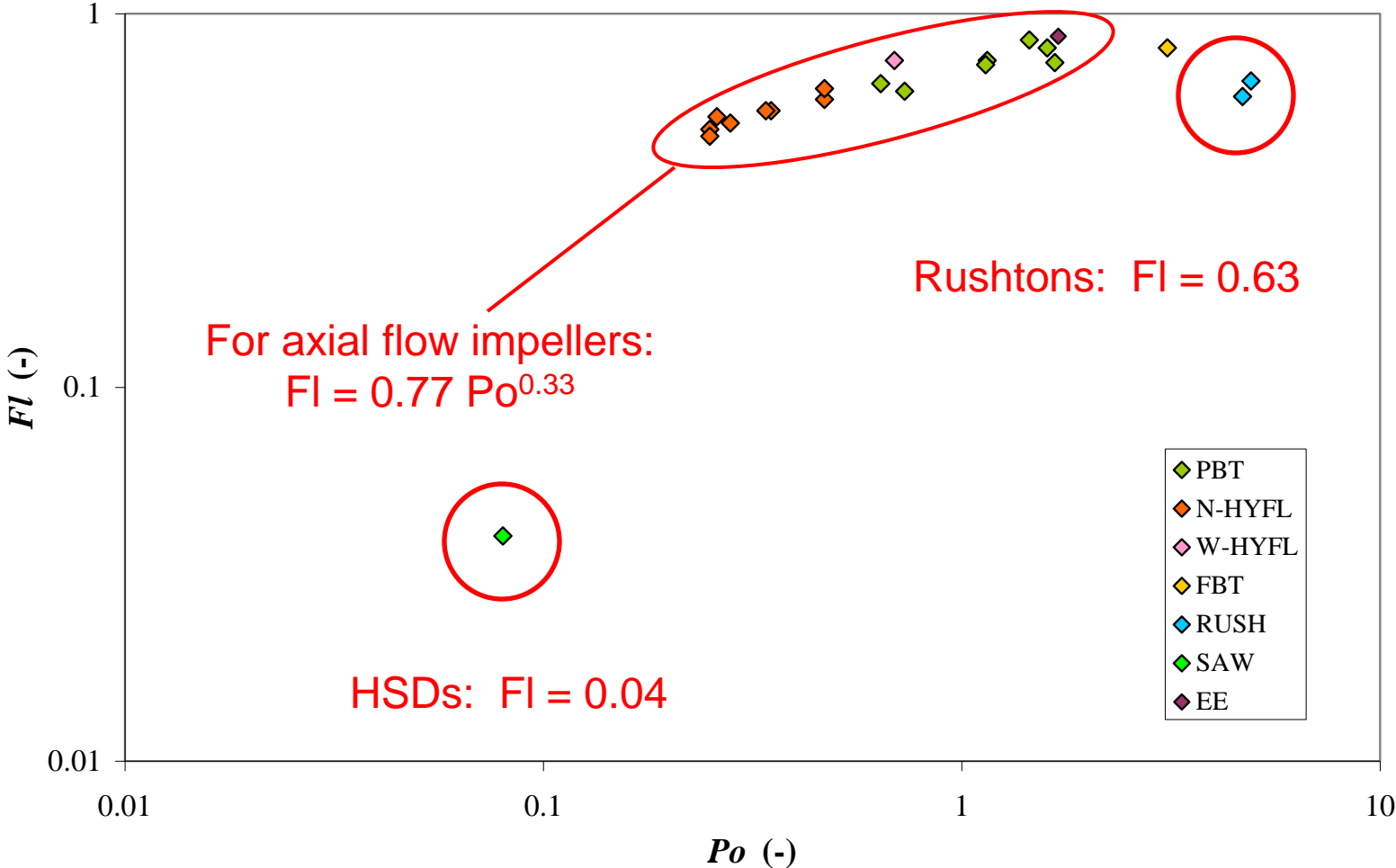


Fig. 4 Near-instantaneous DPIV velocity fields: (a) symmetric; (b) near symmetric with less structure; (c) and (d) asymmetric

Myers et al, *J. Fluids Eng.*, 1997

PLOT FI vs. Po



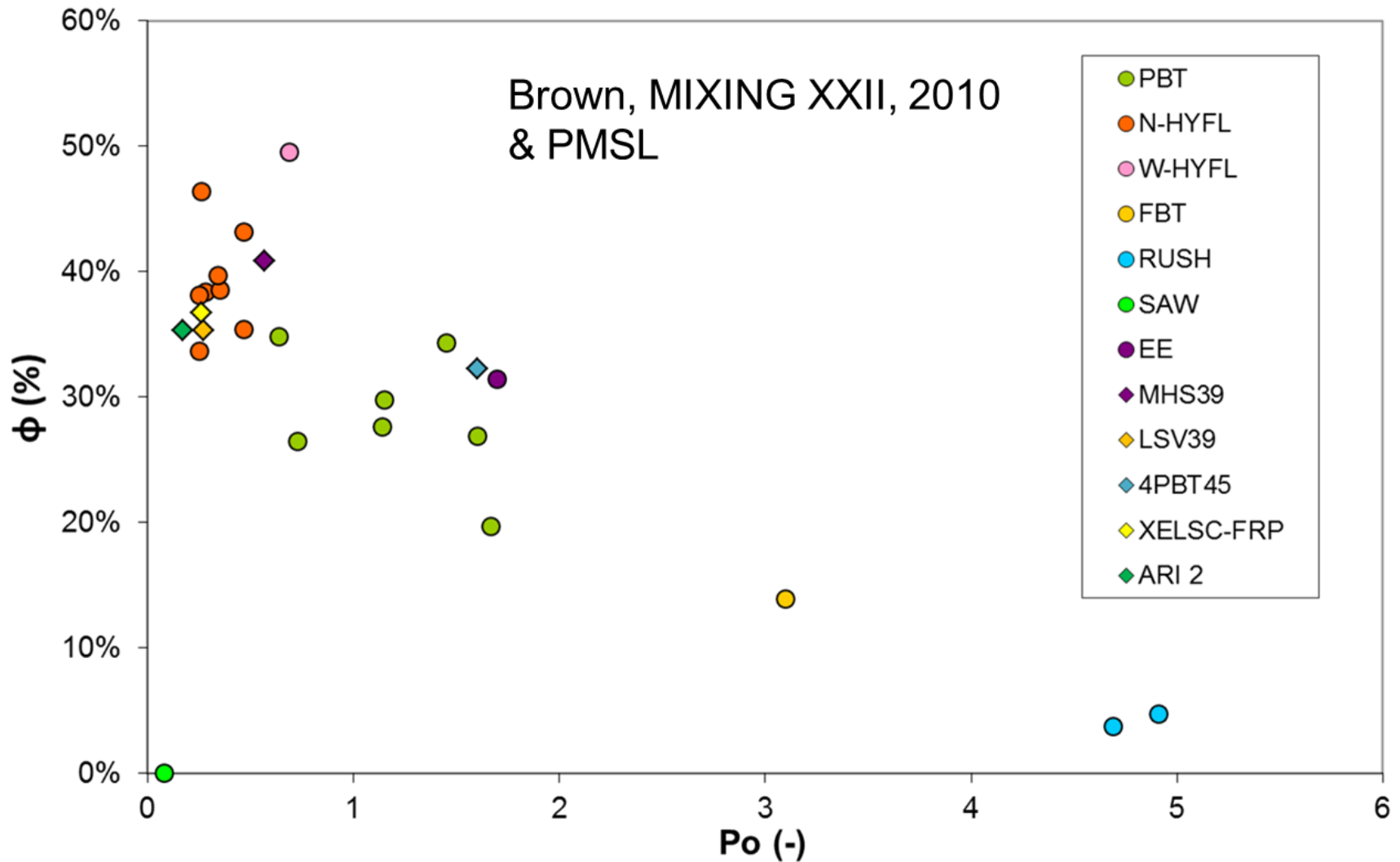
PRESSURE DROP / HEAD

- What is pressure drop in an agitated vessel? What quantity is equivalent to pressure drop in pipe flow?

$$\Delta H(\text{N/m}^2) = \frac{P(\text{W})}{Q(\text{m}^3/\text{s})} \rightarrow \frac{\text{J}}{\text{m}^3} \text{ or } \frac{\text{Nm}}{\text{m}^3}$$

- In an agitated vessel “ ΔH ” is a measure of the impellers’ efficiency:
 - How much energy is input by the impeller to move a volume of fluid?

PLOT OF EFFICIENCY vs. POWER NUMBER



MASS FLOW EFFICIENCY

- Mass of Fluid Pumped per unit of Energy Input (Fořt et al., 2010, Chem. Proc. Eng.).

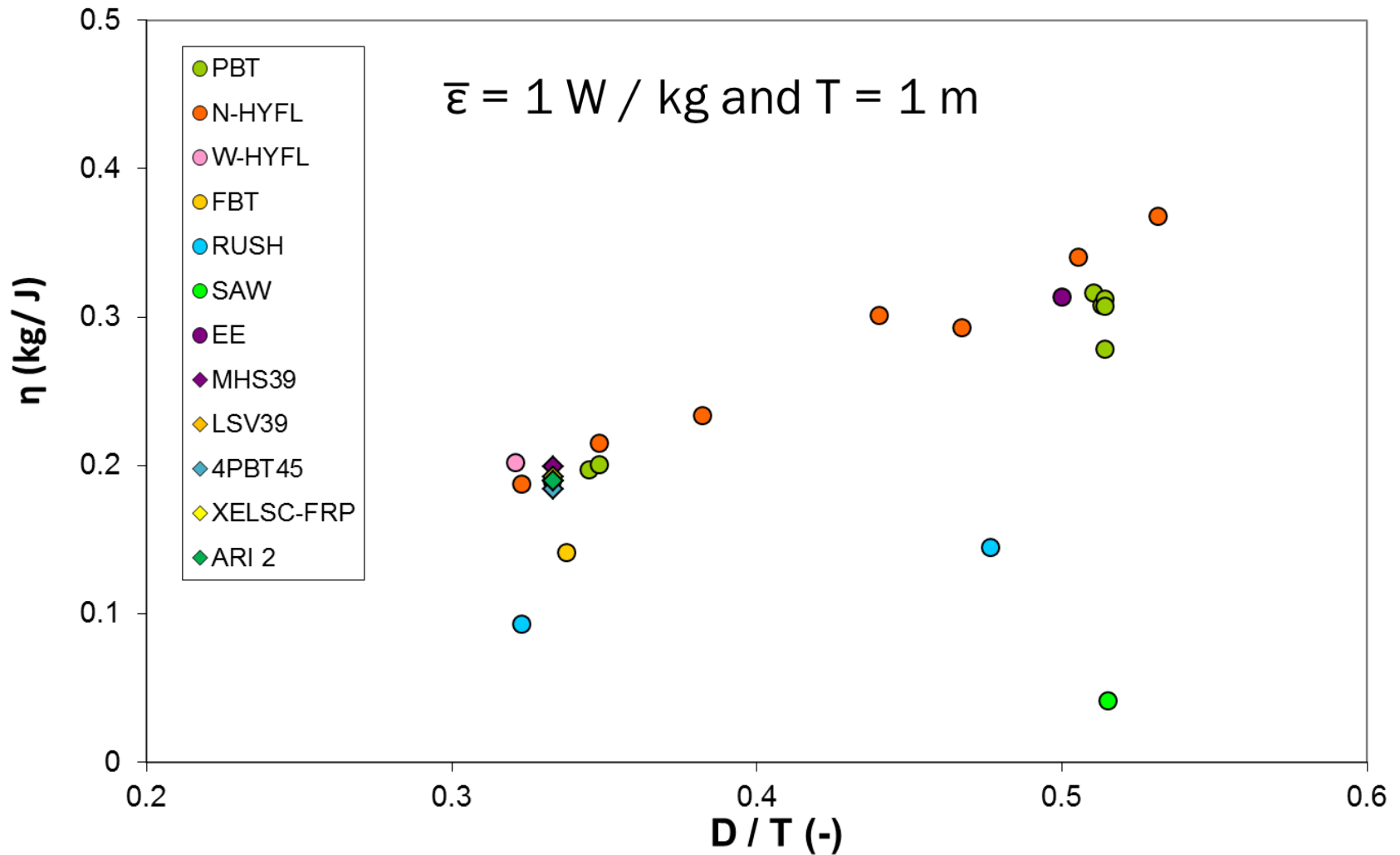
$$\eta = \frac{Fl\rho ND^3}{Po\rho N^3 D^3} = \frac{Fl}{PoN^2 D^2}$$

- At equal Power Input per Mass:

$$\eta = 1.18 \frac{Fl}{Po^{1/3}} \left(\frac{D}{T}\right)^{4/3} (\overline{\epsilon T})^{-2/3}$$

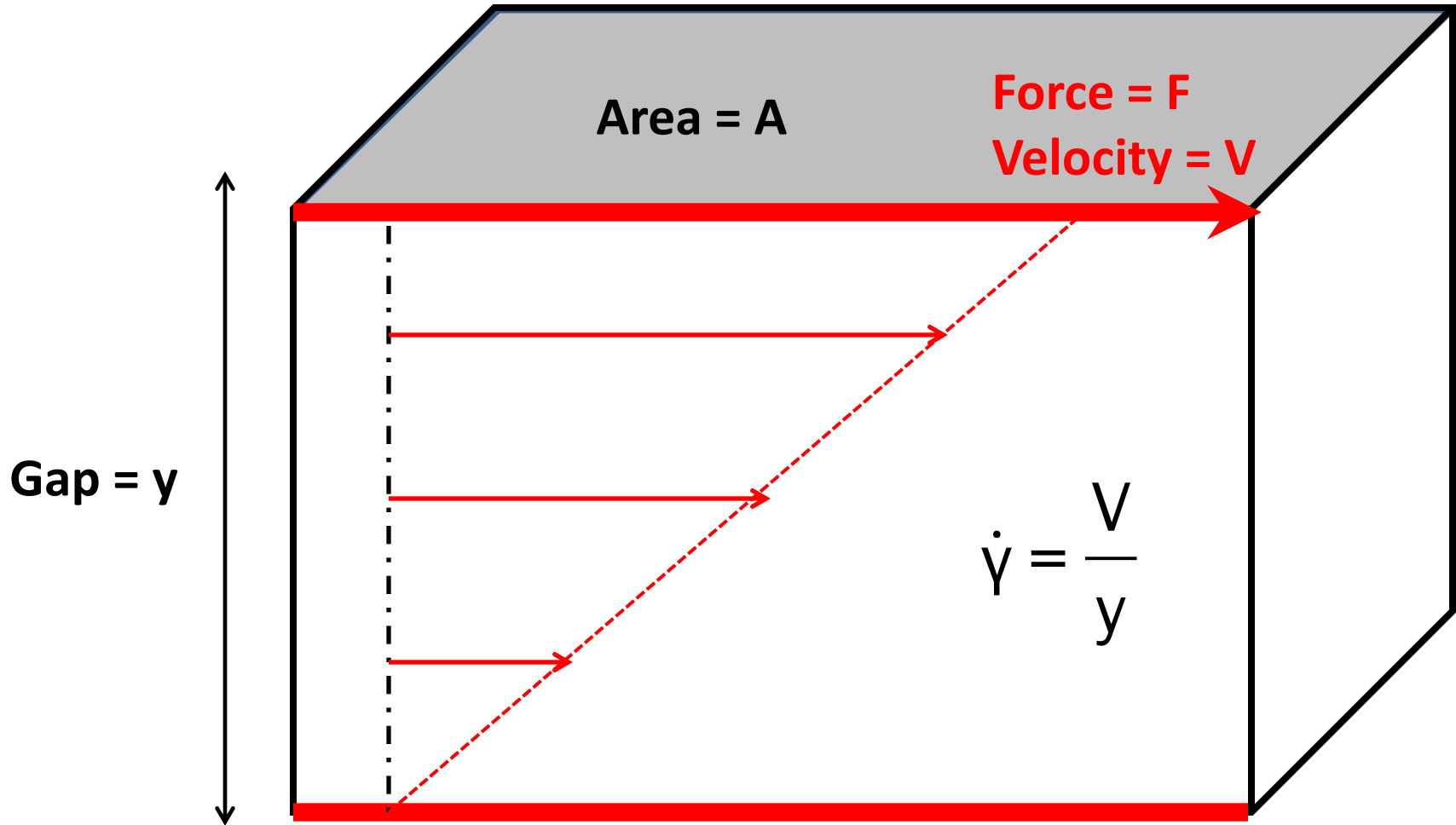
- Plot η versus D/T .

FLOW EFFICIENCY

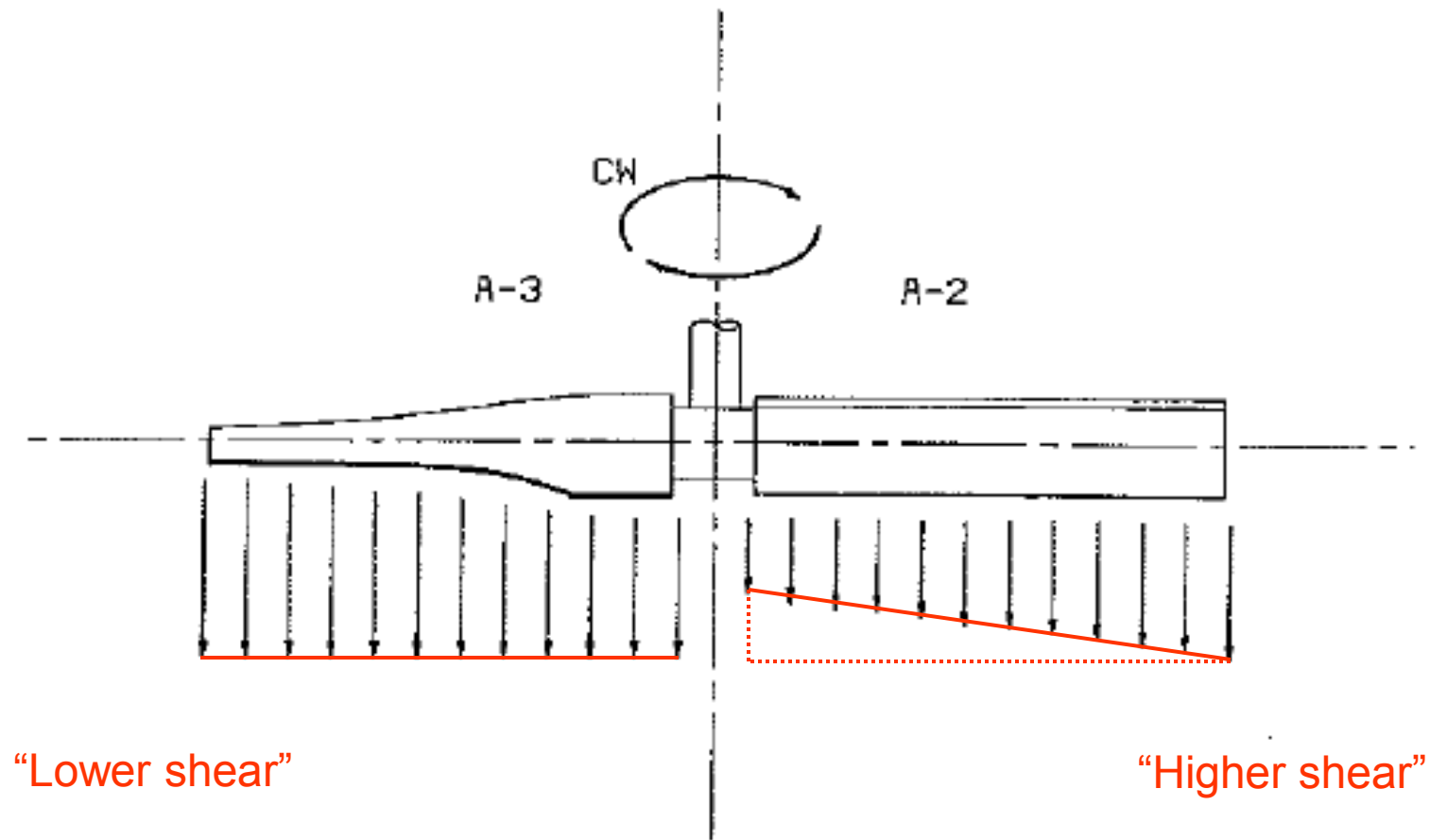


Brown, 2010, MIXING XXII and PMSL

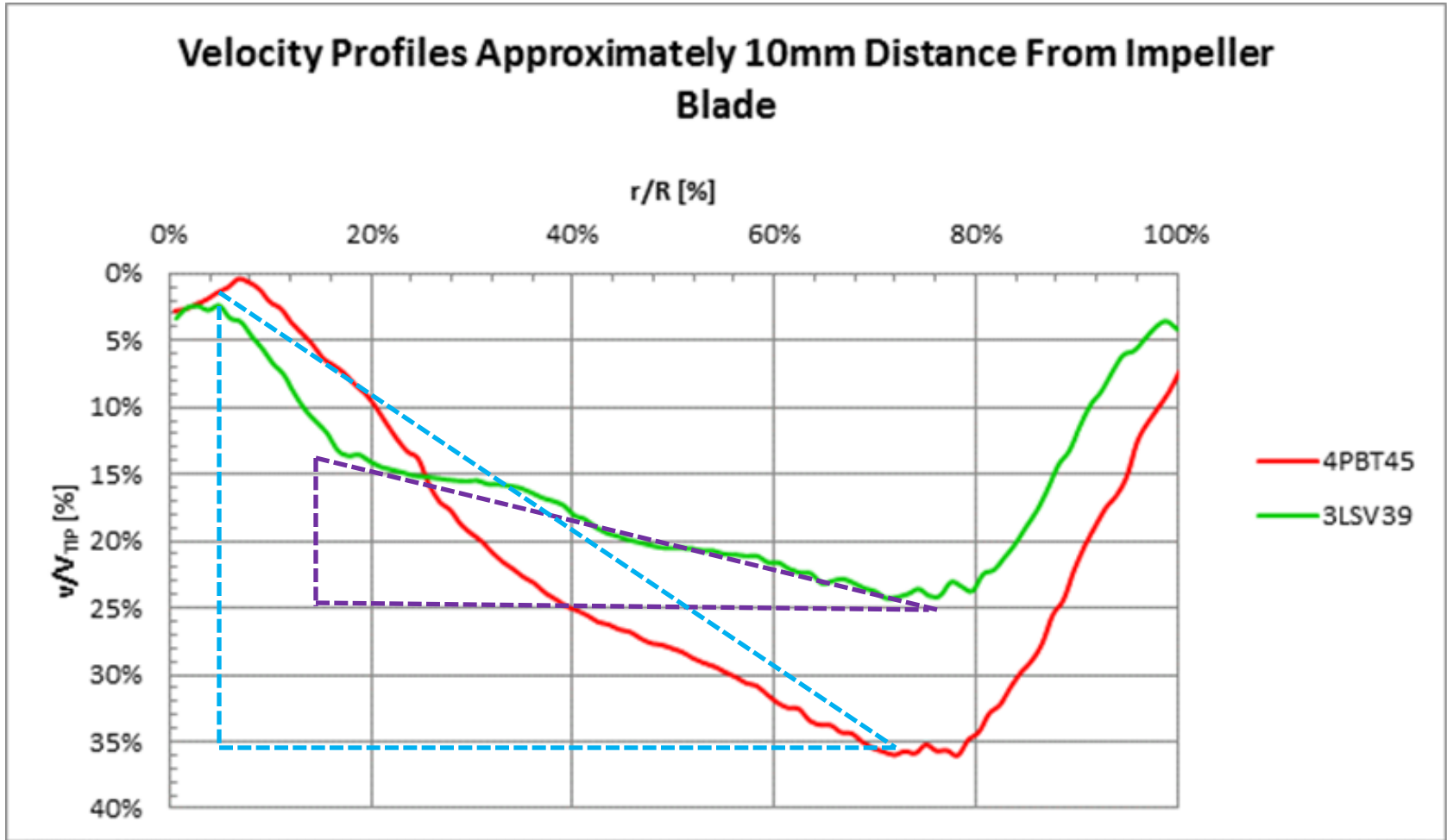
SHEAR RATE



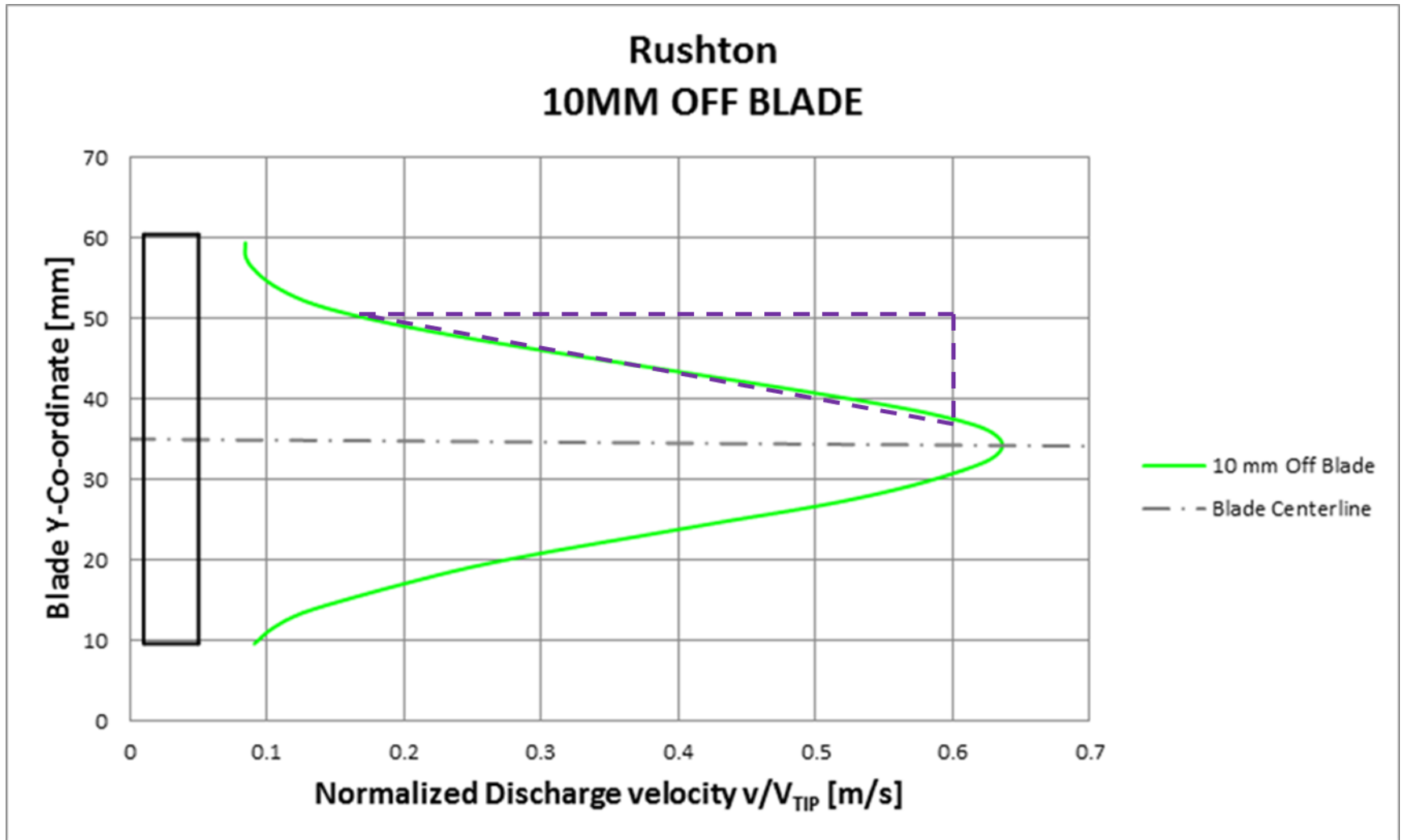
SHEAR (OLDSHUE (1983))



SHEAR (after OLDSHUE (1983))



SHEAR (after OLDSHUE (1983))



SHEAR RATE

- Shear Rate is a Velocity Gradient:
 - Steeper gradient → higher shear rate.

$$\dot{\gamma} = \frac{(\alpha - \beta)}{(\psi - \omega)} \frac{V_{TIP}}{R} = \Lambda \frac{V_{TIP}}{R}$$

Impeller	α	β	ψ	ω	Λ	$\Lambda / \Lambda_{HYDFL}$ at equal V_{TIP}	$\Lambda / \Lambda_{HYDFL}$ at equal $\bar{\epsilon}$
Hydrofoil	0.25	0.14	0.80	0.16	0.17	1.00	1.00
PBT	0.35	0.07	0.65	0.12	0.53	3.11	1.91
Rushton	0.60	0.16	0.12	0.04	5.50	24.4	9.55

PROCESS RESULT

- Dispersion of Silicone Oil in Water.
- Low volume fraction → non-coalescing:
 - Droplet size determined by break-up (Hinze, AIChEJ, 1955).
- Diameters of all impellers are ~50 % of vessel diameter:
 - Rushton turbine.
 - Pitched blade turbines – 45 and 60 degree blade angle.
 - Hydrofoils.
 - Sawtooth / High Speed Dispenser.
- See also Pacek et al., 1999, Chem. Eng. Sci.

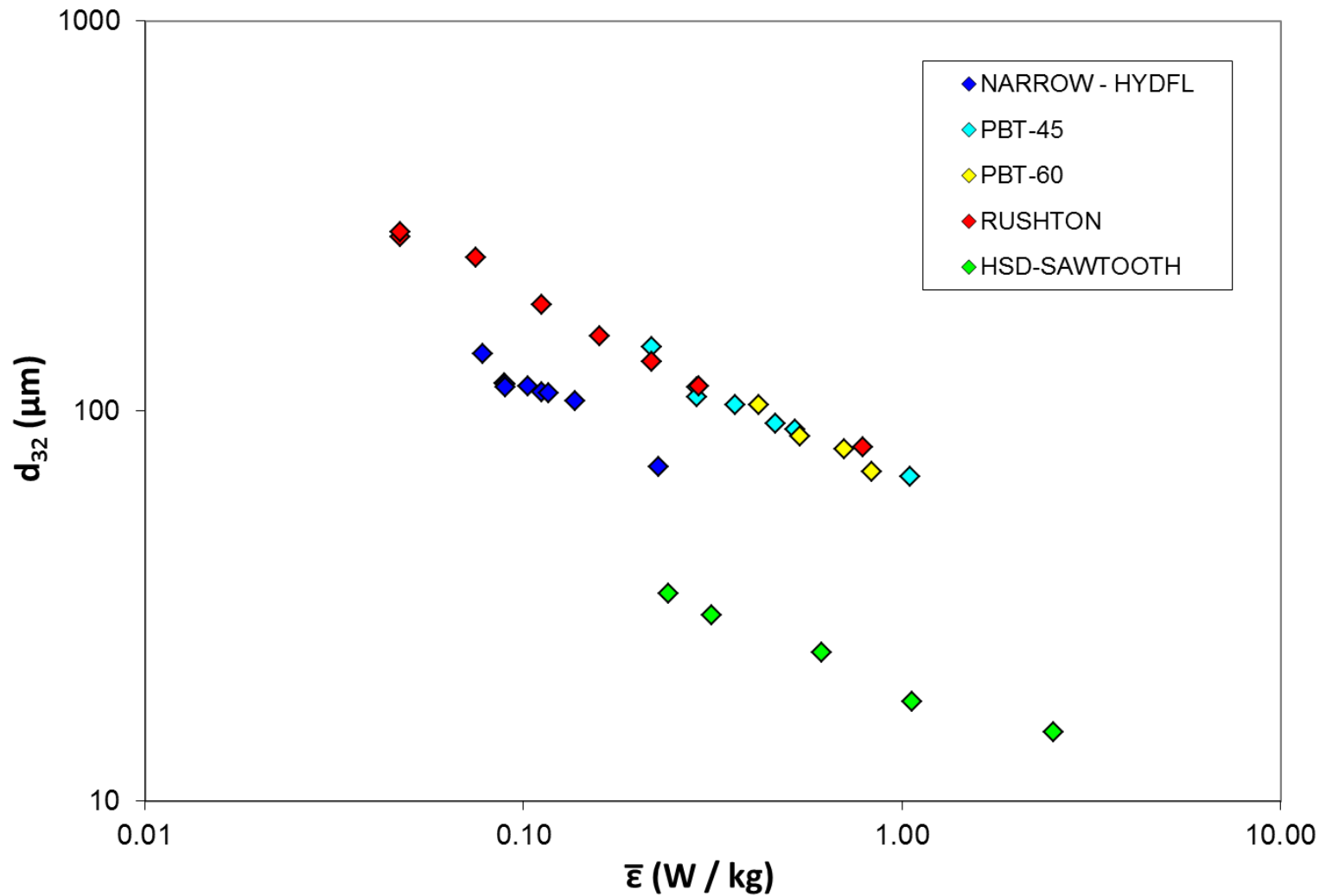
MULTI-PHASE MIXING

- Turbulent eddies interact with second phase.
- Equate stress / energy of turbulent eddy with resisting stress / energy.
- For example, in liquid-liquid and gas-liquid dispersions (Hinze, 1955, AIChEJ):

$$\rho(u')^2 = k_1 \mu \frac{u'}{d} + k_2 \frac{\sigma}{d} \qquad (\epsilon d)^{2/3} d = k_2 \frac{\sigma}{\rho}$$

$$d = k_3 \left(\frac{\sigma}{\rho} \right)^{3/5} \epsilon^{-2/5}$$

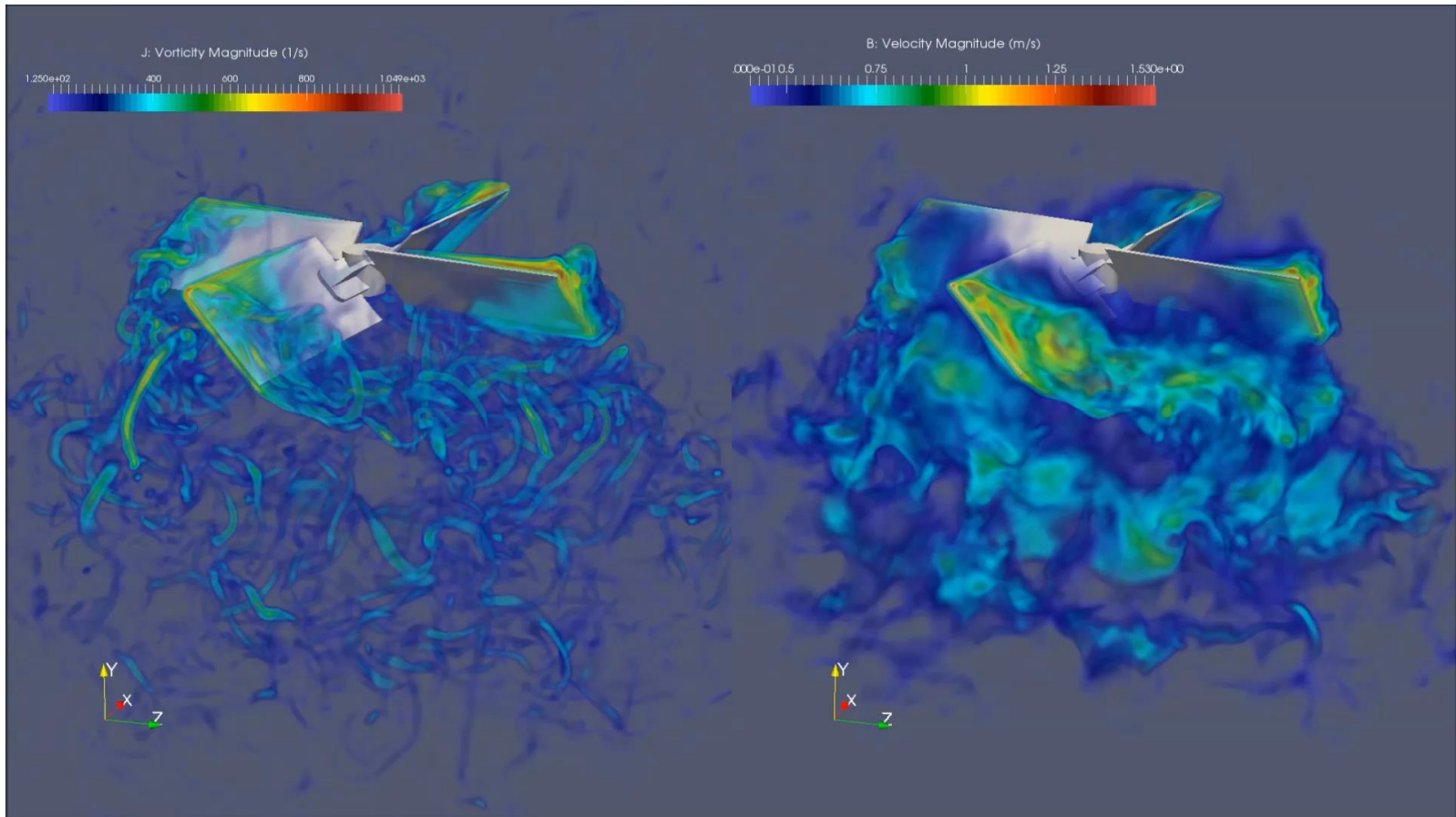
d_{32} vs. $\bar{\epsilon}$ – All $D = \sim T / 2$



RESULTS

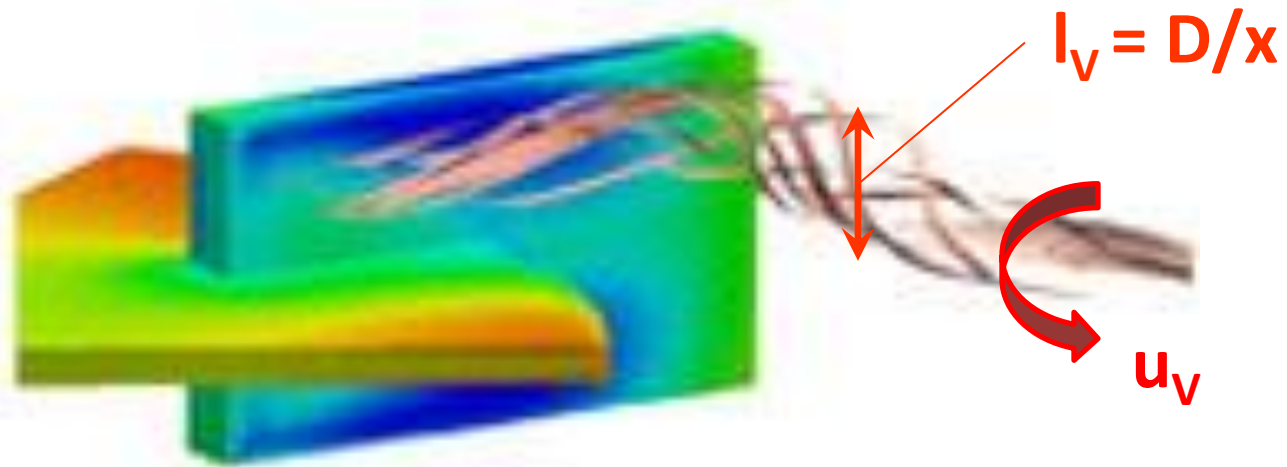
- At the same Power Input per Mass:
 - Rushton and PBTs make the same droplet size.
 - Hydrofoil makes a smaller droplet (~1/2 diameter).
 - HSD / Sawtooth makes the smallest (~1/3 diameter).
- Expect PBT to make smaller drops than Hydrofoil if Hydraulic and Flow Efficiencies and Oldshue's Shear Rate correctly account for impellers' "characteristics".
- What is missing?

TRAILING VORTEX



Lattice Boltzmann Large-Eddy Simulation using DMT software from M-Star Simulations

TRAILING VORTEX



Turbines: $I_v \approx w_p / 2$
Hydrofoils: $I_v \approx w_p$

<http://www.csiro.au/~media/34F42F86D14645F3B6C3710F4306392E.ashx>

AVERAGE POWER PER MASS


- Simple calculation if impeller Power number is known.
- For vessel where $H = T$:

$$\bar{\varepsilon} = \frac{PoN^3D^5}{(\pi/4)T^3}$$

MAXIMUM TKE, k_{\max} , & TKEDR, ϵ_{\max}

- Trailing Vortex Kinetic Energy is scaled by Tip Speed Squared.
- Grenville et al. (2008) showed that, for bladed impellers:

$$\frac{k_{\max}}{U_{\text{TIP}}^2} = 0.104Po^{1/2} \quad (\pm 10\%)$$

$A = 1$ 

$$\epsilon_{\max} = A \frac{k_{\max}^{3/2}}{l_v} = Ax \frac{k_{\max}^{3/2}}{D} \quad \text{where: } l_v = \frac{D}{x}$$

VORTEX LENGTH SCALE – I_v

- Measurements made using Dantec three beam, two component LDA in back scatter mode with Burst Spectrum Analyzer (e.g. Lee & Yianneskis (1998)).

$$I_v = \frac{D}{x}$$

Rushton turbine:	$x = 12$
Pitched blade turbine:	$x = 17$
A310 Hydrofoil:	$x = 16$
Sawtooth impeller:	$x = 12$

TVEDR

- Combining:

$$k_{\max} = 0.104Po^{1/2}U_{\text{TIP}}^2$$

$$\varepsilon_{\max} = Ax \frac{(0.104Po^{1/2}U_{\text{TIP}}^2)^{3/2}}{D}$$

$$\varepsilon_{\max} = 1.04xPo^{3/4}N^3D^2$$

RATIO

- Ratio of maximum TVEDR to average Power per Mass:

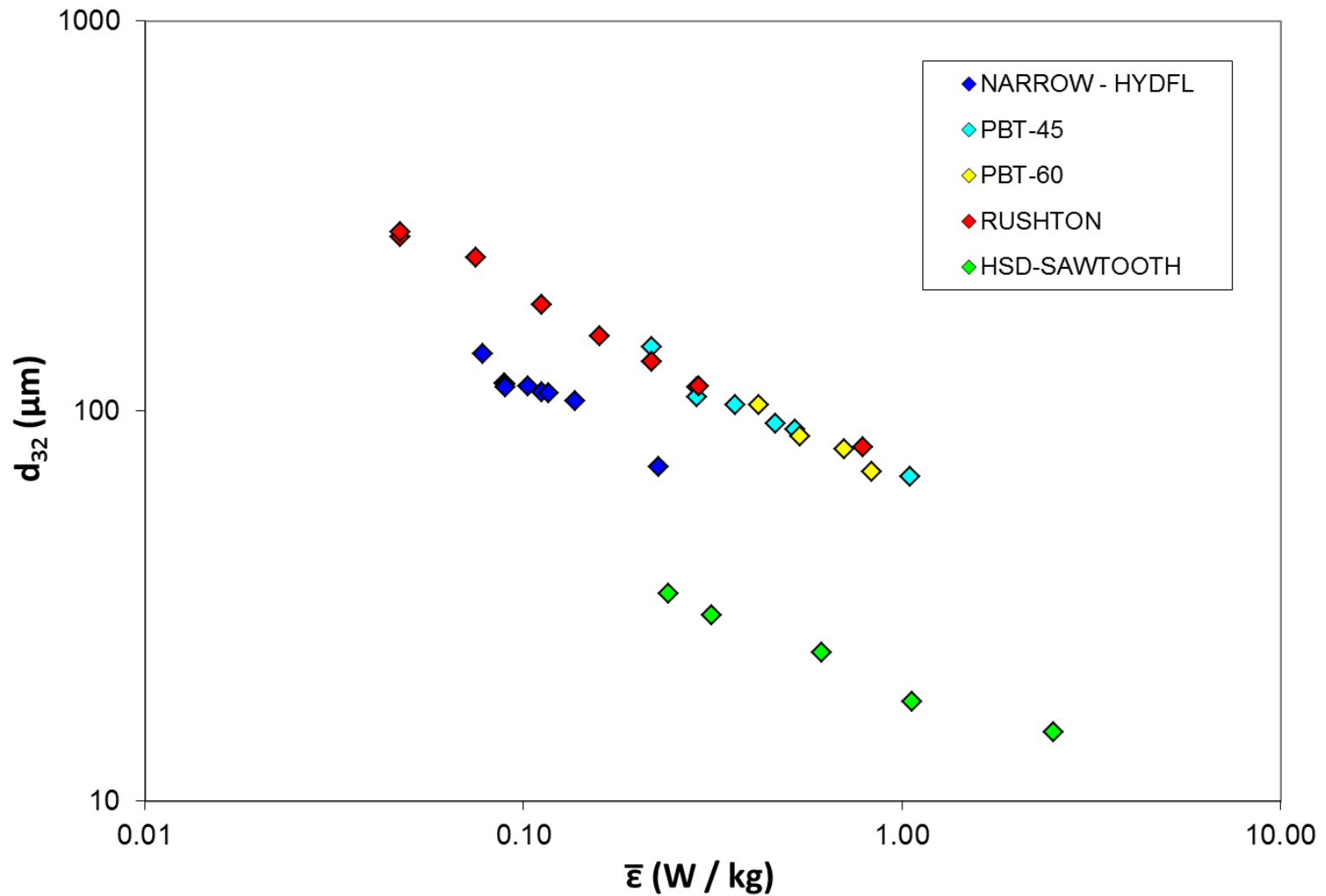
$$\frac{\epsilon_{\max}}{\bar{\epsilon}} = 1.04 \times P_o^{3/4} N^3 D^2 \times \frac{(\pi/4) T^3}{P_o N^3 D^5}$$

$$K = \frac{\epsilon_{\max}}{\bar{\epsilon}} = 0.82 \frac{x}{P_o^{1/4}} \left(\frac{T}{D}\right)^3 (\pm 15\%)$$

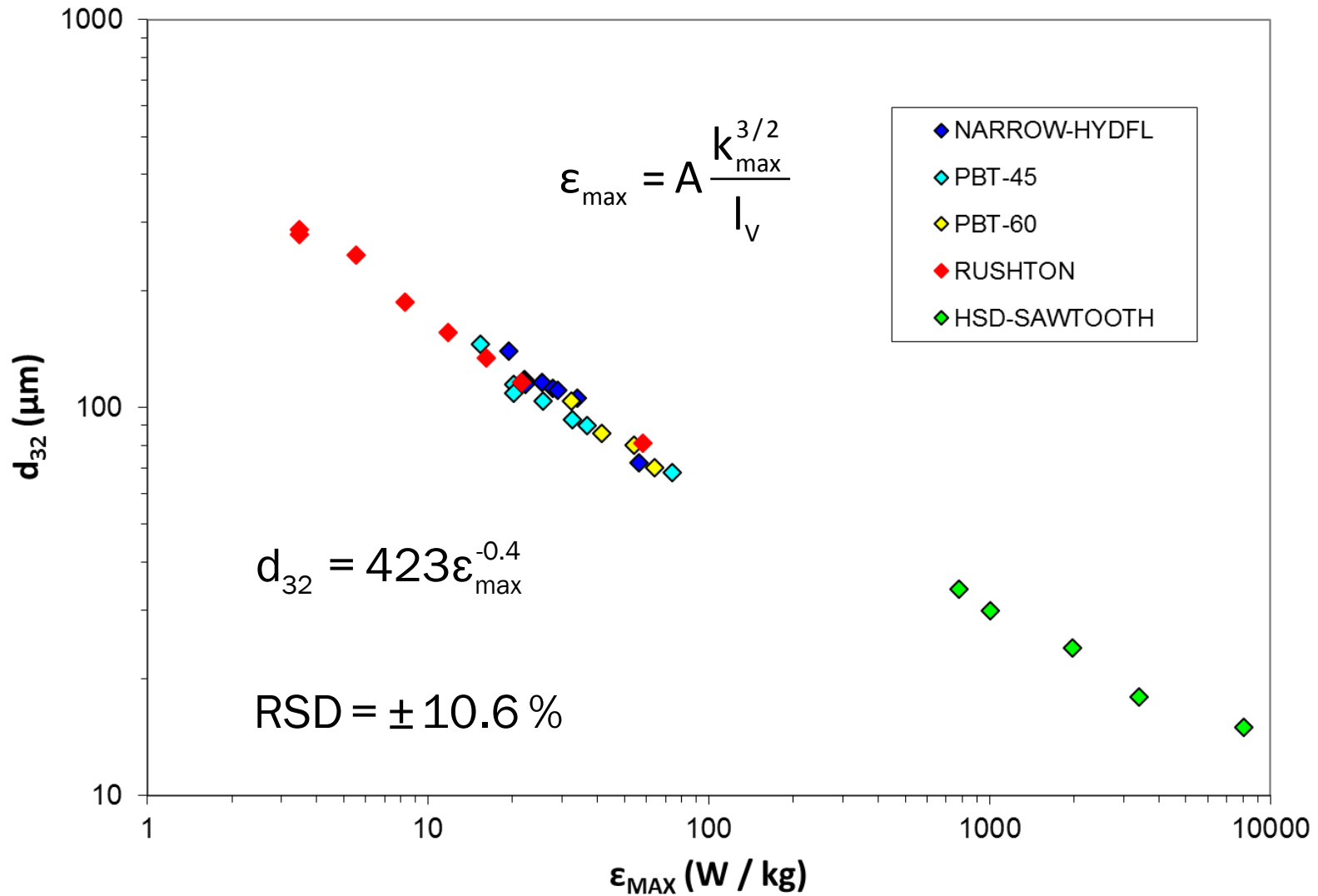
$$\Phi = \frac{K}{K_{\text{Rushton}}}$$

Impeller	x	Po	D / T	K	Φ
Rushton	12	4.69	0.48	70.43	1.00
4PBT-45	17	1.16	0.51	75.08	1.07
4PBT 60	17	1.67	0.51	83.58	1.19
A310	16	0.26	0.44	258.6	3.67
Sawtooth	12	0.08	0.51	3318	47.11

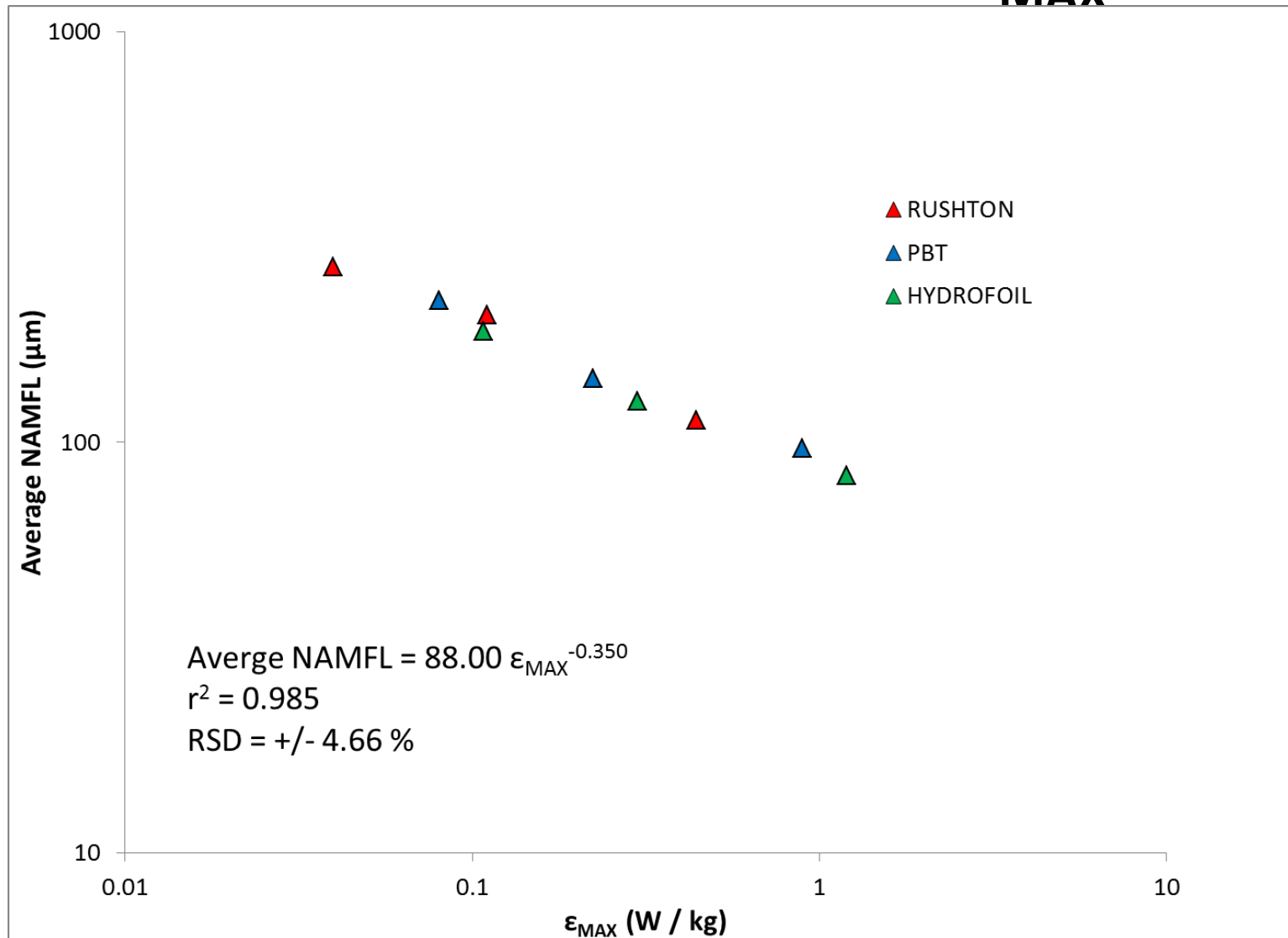
d_{32} vs. $\bar{\epsilon}$ – All $D = \sim T / 2$



d_{32} versus $\epsilon_{\max} - D = \sim T/2$



AVG. FLOC LENGTH vs. ϵ_{MAX}



CONCLUSIONS

- Impellers in stirred tanks are pumps:
 - Power input and flow generated can be measured.
- Impeller characteristics are rigorously quantified:
 - Hydraulic and Flow Efficiencies.
 - Maximum Energy Dissipation Rate in Trailing Vortex.
- Characterization confirmed by Equilibrium Droplet and Floc Sizes generated with different impellers.
- “Shear”:
 - Not strictly correct mechanistic description.
 - Conveys concept → size reduction or growth.
- Do we mean “high stress impellers”?
- Need to question “conventional wisdom” – not just for mixing:
 - Better measurement, analysis and design techniques.

BACK-UP CHARTS

TURBULENT & LAMINAR REGIMES

- In turbulent regime: $X = \frac{\pi fL}{2D} = \text{Constant}$

$$P = X\rho U^3 D^2$$

- In laminar regime:

$$f = \frac{16}{\text{Re}}$$

$$X = \frac{\pi}{2} \frac{16}{\text{Re}} \frac{L}{D} = \frac{Y}{\text{Re}}$$

$$P = Y\mu U^2 D$$

ENERGY & POWER

- Kinetic energy $\rightarrow m * U^2$
- Kinetic energy per volume $\rightarrow \rho * U^2$
- Kinetic energy per mass $\rightarrow U^2$
- Characteristic timescale $\rightarrow L / U$
- Power per mass = Energy per mass / Time $\rightarrow U^3 / L$
- Also “energy dissipation rate (per mass)”.

POWER INPUT PER UNIT MASS

- Power Input to fluid:

$$P = \frac{\pi fL}{2D} \rho U^3 D^2$$

- Mass of fluid in pipe:

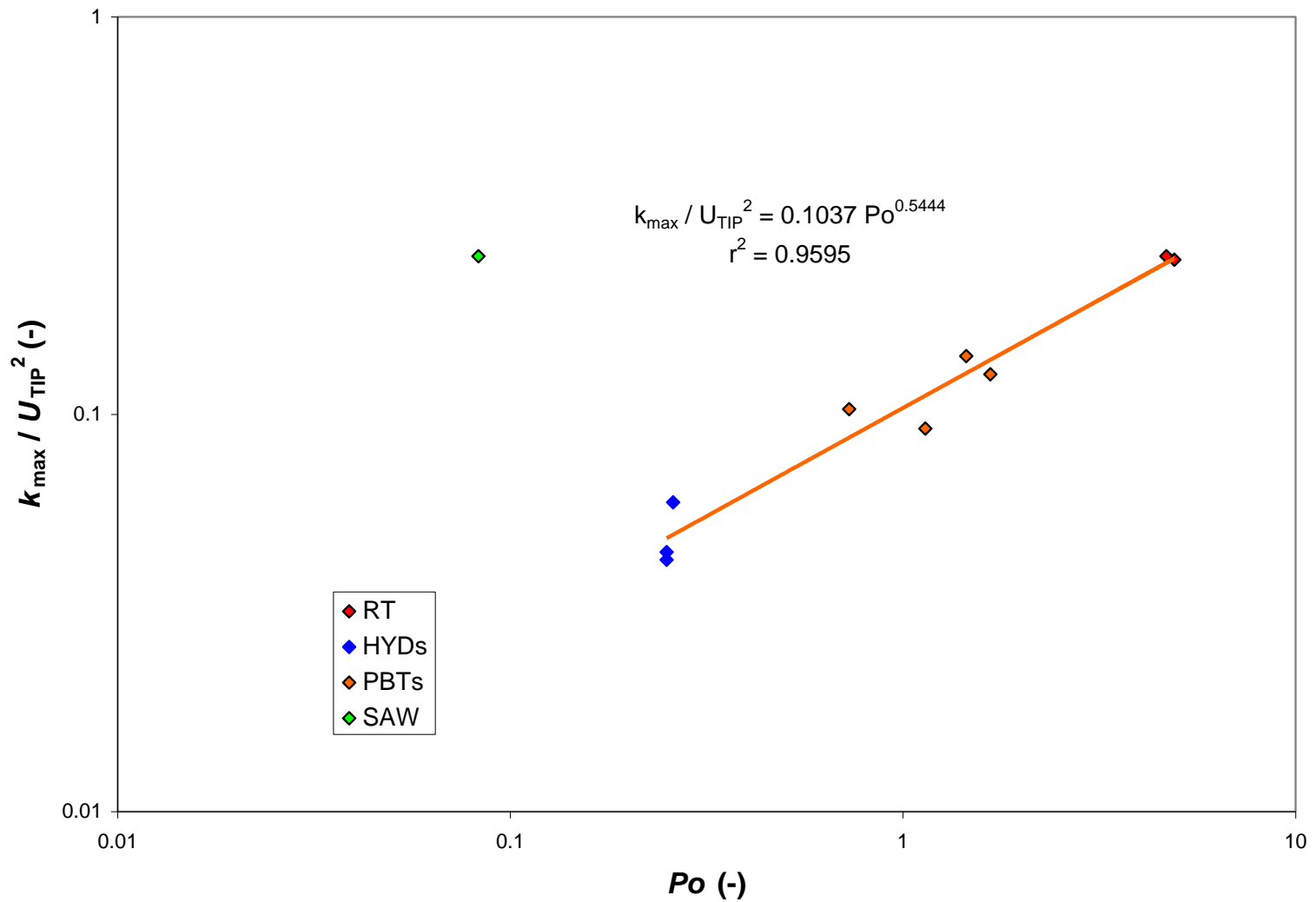
$$M = \frac{\pi}{4} \rho L D^2$$

- Power Input per Mass:

$$\bar{\varepsilon} = \frac{P}{M} = \frac{(\pi/2)fL\rho U^3 D}{(\pi/4)\rho L D^2} = 2f \frac{U^3}{D}$$

- Characteristic Velocity and Length Scale for pipe flow.

GRENVILLE et al. (2008)



FLOCCULATION

- Removal of fine particles (sub-micron) from liquid:
 - Water and Waste Water Treatment.
 - Polymer processing.
- Flocculation increases particle size:
 - Agglomerates of many particles.
 - Easier to remove by sedimentation or filtration.
- Final floc size determined by equilibrium between growth and breakage.
- Time affects floc shape and density:
 - Breakage and re-flocculation.
- Effect of mixing / shear rate?

LITERATURE

Stein & Camp, 1943, J. Bos, Soc. Civ. Engrs.

Defined Shear Rate – G.

$$G = \left(\frac{\bar{\varepsilon}}{\nu}\right)^{1/2} = \left(\frac{P}{V\mu}\right)^{1/2} \quad \left[= \frac{1}{\tau_K}\right]$$

Commonly used in Water Treatment industry.

Scale-up of Jar Tests.

http://www.nesc.wvu.edu/pdf/dw/publications/ontap/2009_tbjar_testing_dwfsom73.pdf

Benz, 2007, Chem. Eng. Prog.

“G-value has no legitimate use in designing or specifying agitators”.

LITERATURE

Koh et al., 1983, Chem. Eng. Sci.

Defined effective Shear Rate:

Couette flow → Laminar and Taylor Vortices:

$$G_{\text{Eff}} = (G_{\text{Lam}}^2 + G_{\text{Vort}}^2)^{1/2} \quad G_{\text{Vort}} = \left(\frac{\varepsilon_{\text{Vort}}}{\nu}\right)^{1/2}$$

Coufort et al., 2007, Chem. Eng. Proc.

For both inertial and viscous sub-range expect:

$$L_{\text{Floc}} \propto \varepsilon^{-1/4}$$

$\bar{\varepsilon}$ not sufficient to predict agglomeration phenomena.

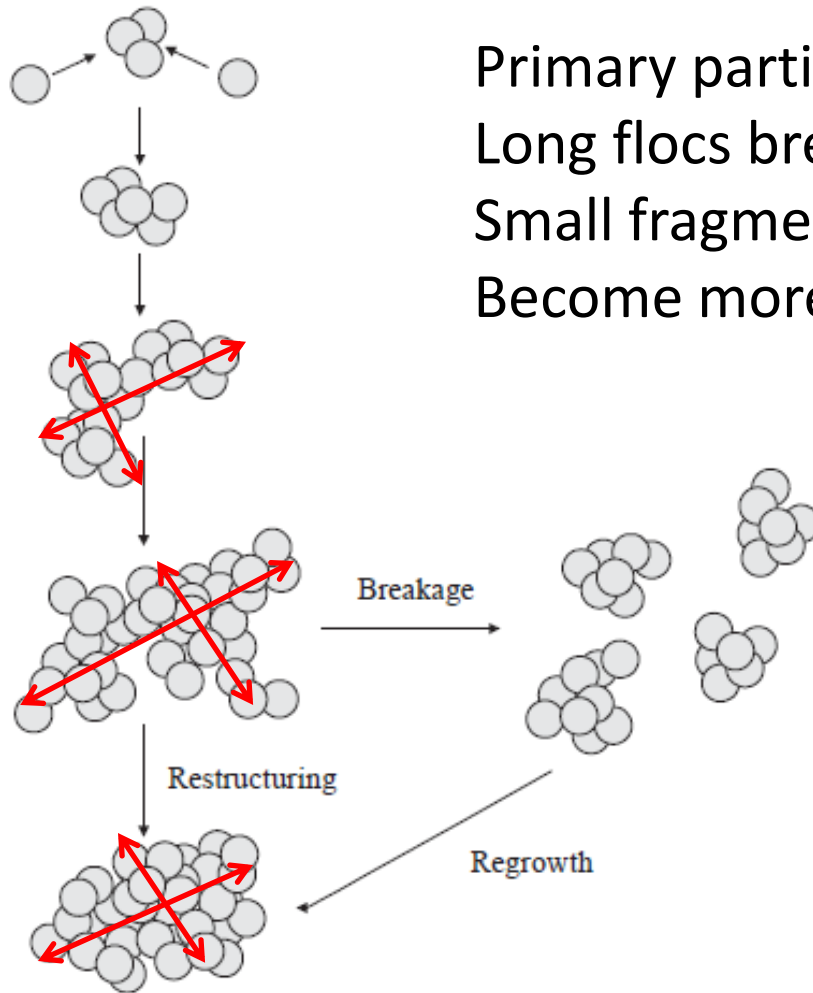
Knowledge of local hydrodynamics is needed.

Kresta, 1998, Can. J. Chem. Eng.

SPICER, 1997, PhD THESIS

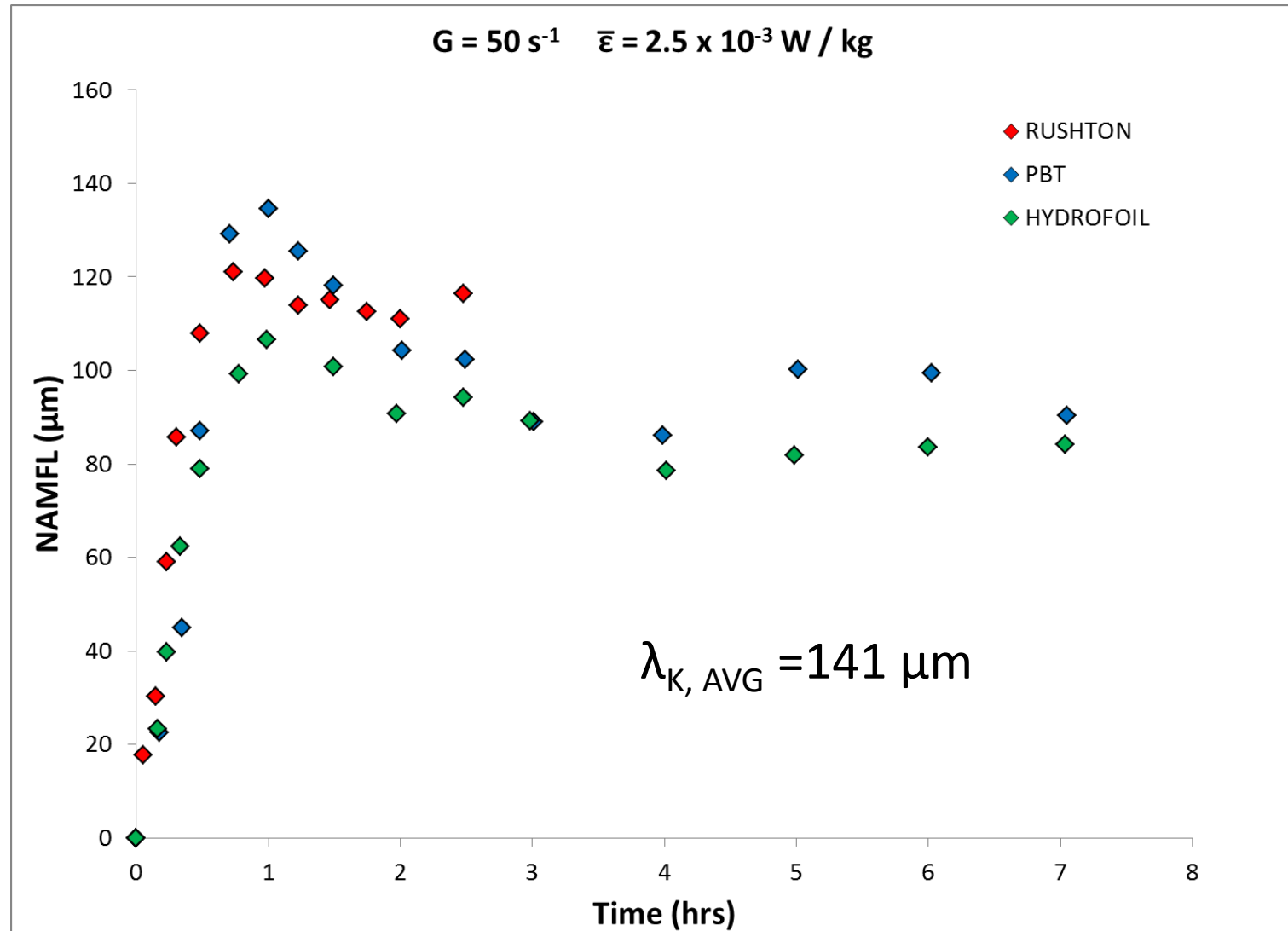
- Measured Floc Length versus Time:
 - Equipment:
 - Tank: $T = 0.15 \text{ m}$
 - Impellers: Rushton, PBT and Hydrofoil $D = T/3$
 - G-value: 15, 25 and 50 s^{-1}
 - Particles: Polystyrene $d = 870 \text{ nm}$ $\phi_V = 1.4 \times 10^{-5}$
 - Flocculant: Aluminum sulfate hydrate 0.5 g / lt.
 - Initially mixed at $G = 300 \text{ s}^{-1}$ to remove agglomerates.
- FSD measured with optical microscope and video camera.
 - Images digitized and recorded with Image Analysis software.
- Plot Number Average Maximum Floc Length versus Time:
 - Longest dimension of Floc.

MECHANISM OF FLOC GROWTH

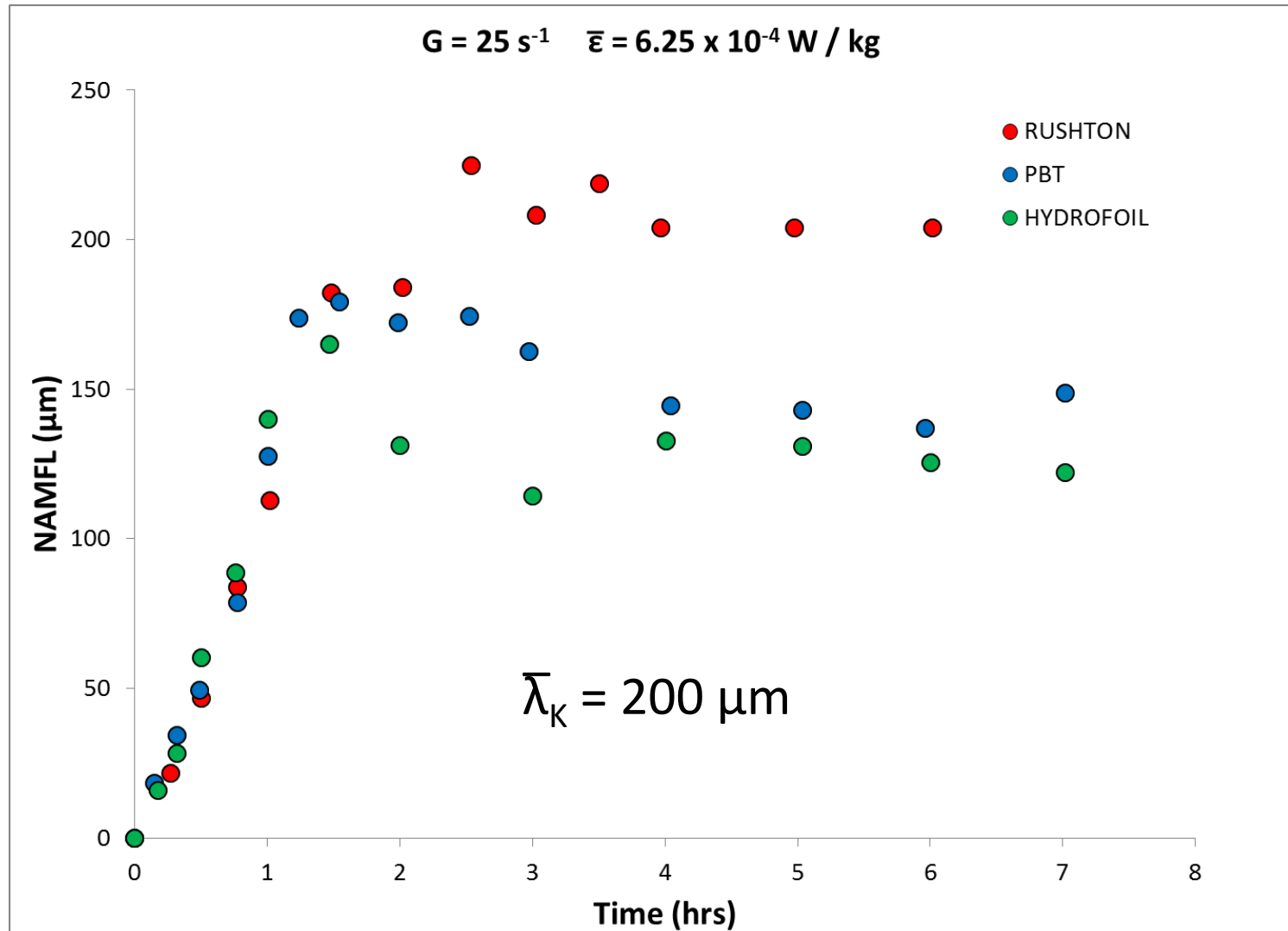


Primary particles collide and agglomerate.
Long flocs break.
Small fragments collide and reorganize.
Become more compact over time.

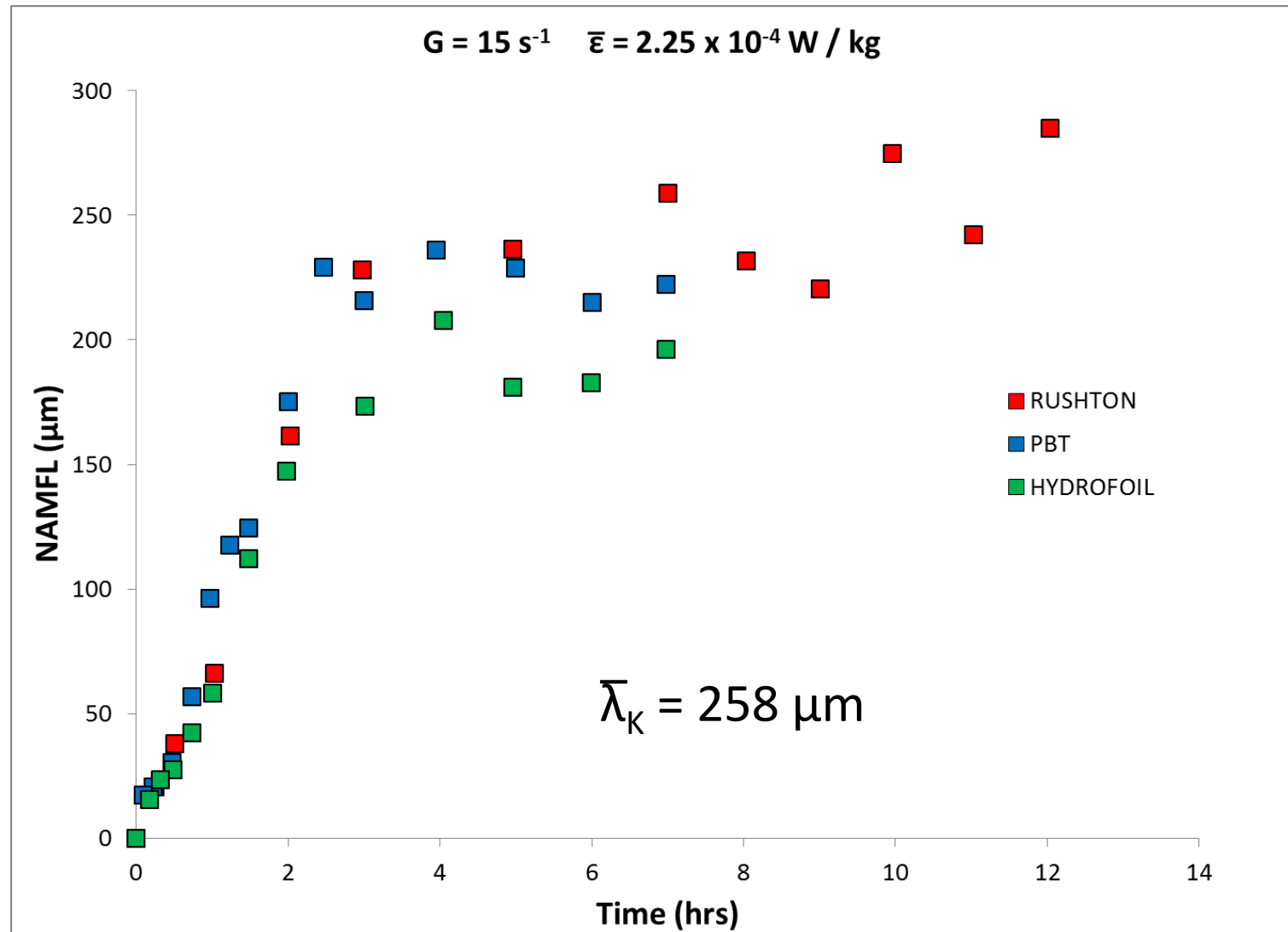
NAMFL vs. TIME



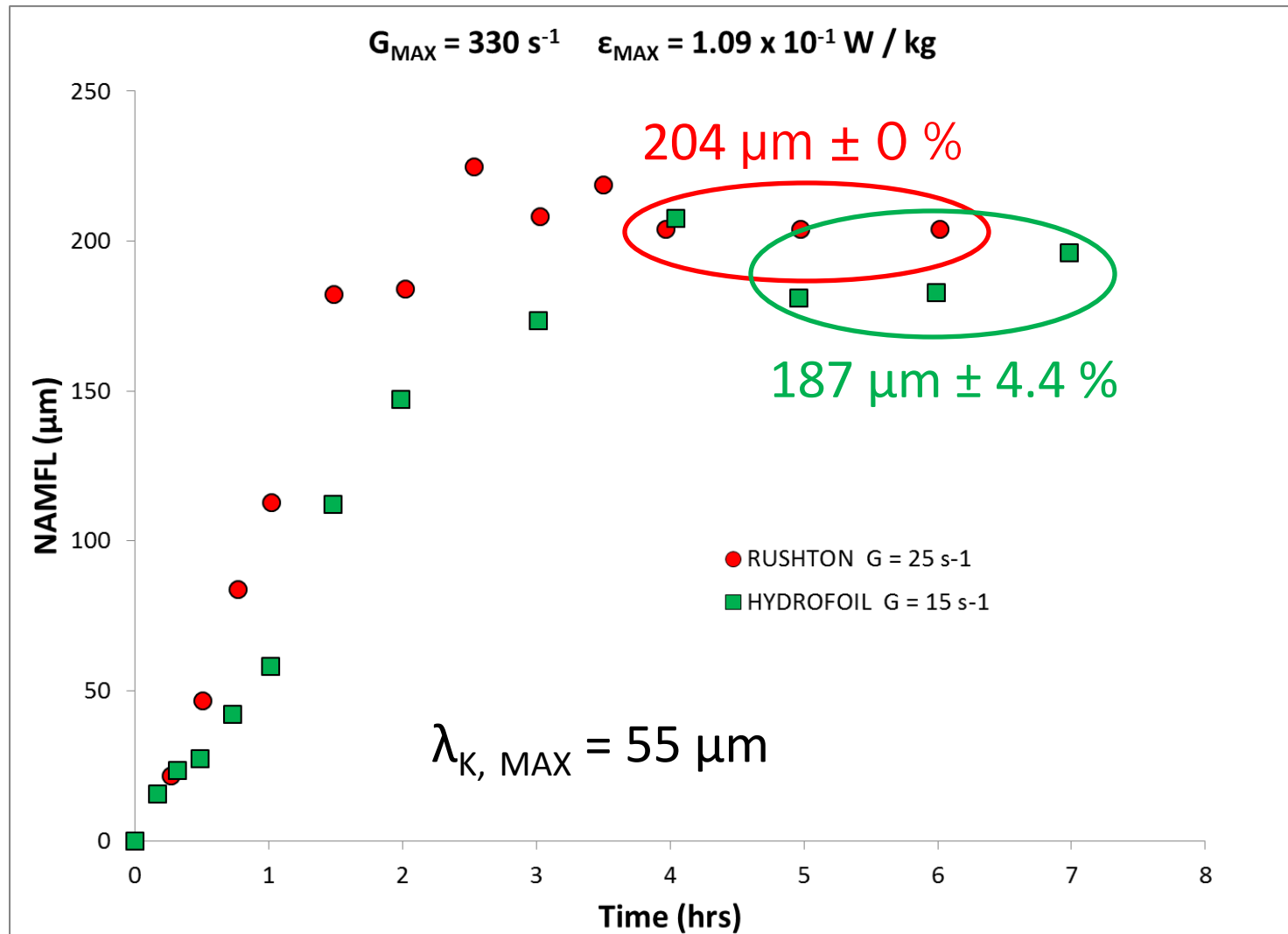
NAMFL vs. TIME



NAMFL vs. TIME



NAMFL vs. TIME – EQUAL ϵ_{MAX}



EXPONENT $x = -0.35$

- Exponent, x , on ε_{MAX} is -0.35:
 - Upper 95 % CL = -0.313
 - Lower 95 % CL = -0.386
 - t-Stat = -22.72
 - P-value $\lll 0.05$
- Coufort et al., 2007: $x = -0.250$
 - Floc size in inertial and viscous sub-ranges.
- Shinnar, 1961 (and others): $x = -0.250$
 - Coalescence controlled liquid-liquid dispersions.
- Calabrese et al., 1986 (and others): $x = -0.400$
 - Break-up controlled liquid-liquid dispersions.
- Schulze et al., 2000 (and others): $-0.250 > x > 0.400$
 - Dependent on balance between break-up and coalescence.
- Equilibrium floc size dependent on break-up and growth.

WHAT DO WE MEAN BY SHEAR?

- Tilton, 2016, Private Communication:
 - Shear (high or low) is generally used terminology:
 - Conveys the general concept – (in)efficient disperser.
 - Not strictly the best mechanistic description.
 - Need Energy dissipation rate in region where desired mixing process is controlled:
 - $\bar{\epsilon}$ for “global” processes, e.g. blend time.
 - ϵ_{MAX} for “local” processes, e.g. drop break-up, flocculation, micro and mesomixing (using a dip pipe).
- High shear impeller \rightarrow high U_{TIP} and small w_p .
- Define “Turbulent (Corrsin) Shear Rate”:

$$\dot{\gamma}_c \sim \frac{u'}{l_0} = \frac{k_{MAX}^{1/2}}{l_0} \rightarrow u' \sim (\epsilon_{MAX} l_0)^{1/3} \rightarrow \dot{\gamma}_c \sim \frac{\epsilon_{MAX}^{1/3}}{l_0^{2/3}} \left[\propto \frac{1}{\tau_c} \right]$$

d_{32} versus $\gamma_C - D = \sim T/2$

