THE EVAPORATION AND DISPERSION OF FLAMMABLE LIQUID SPILLAGES

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Methods are described for calculating the sizes and rates of generation of vapour clouds from spillages of volatile liquids, liquified gases and cryogenic liquids. The equations for their subsequent dispersion, which enable concentrations to be calculated, are given.

INTRODUCTION

The generation of a vapour cloud from the spillage of a volatile liquid and the subsequent dispersion of the cloud down-wind are separate but inter-related problems. The first part of this paper deals with the generation of the cloud; the second with its dispersion.

Although primarily concerned with flammable vapours and the hazards thereof, the principles are equally applicable to the generation and dispersion of other noxious vapours, for example by the spillage of a volatile toxic liquid.

GENERATION OF VAPOUR CLOUDS

The manner in which a vapour cloud is produced depends upon the initial state of the liquid spilled in respect of its temperature and vapour pressure. Three conditions may be distinguished:

(1) A volatile liquid at ambient temperature and pressure, e.g. petrol

(2) A liquid at ambient temperature and an elevated pressure, e.g. liquified gases such as chlorine, butane, etc.

(3) A liquid at a low temperature and at ambient pressure, e.g. a cryogenic liquid.

A fourth condition is of importance in chemical plants, namely a liquid at an elevated temperature and above ambient pressure, that is to say a liquid above its normal boiling point. It will be seen, however, that this may be treated as a variant of the second class.

The differences between the three classes depend upon two things: The initial behaviour at the instant of release and the rate-controlling processes which determine the subsequent rate at which vapour is produced. With the second and third classes there is an immediate flash-off or boil-off on release during which the liquid acquires a steady state equilibrium with its environment. Thereafter vapour is produced at a rate determined by heat transfer conditions. In the case of the first class, when the liquid is in equilibrium with its surroundings, there is no initial flash-off. Vapour generation is determined by its vapour pressure and wind velocity.

Evaporation of a volatile liquid

Considering, first, a volatile liquid, it is assumed that the liquid is spilled into a bund so as to present a constant area. Under all real conditions it is known that the rate at which evaporation takes place is directly dependent upon the rate at which the wind carries the vapour away which in turn depends upon the vapour pressure of the liquid.

This is a diffusion process depending upon the wind velocity. It is relatively slow so that heat transfer from the substrate is more than sufficient to provide the latent heat of vapourisation and there is a negligible fall in temperature of the liquid surface.

Sutton (1, 2) has shown that the evaporation rate from a rectangular surface set square to the wind direction is given by:

$$E(\bar{u}, x, y) = \Lambda \bar{u}^{(2-n)/(2+n)} \cdot x^{3/(2+n)} \cdot y$$

where $E$ is the evaporation rate; $\bar{u}$ is the mean wind velocity; $x$ is the downwind dimension of the surface; $y$ is the crosswind dimension; $\eta$ is a turbulence factor dependent upon meteorological conditions; and $\Lambda$ is a complex function.

A similar equation applies in the case of a circular surface being derived by integrating the evaporation from a series of downwind strips:

$$E(\bar{u}, h) = 3\Lambda \bar{u}^{(2-n)/(2+n)} \cdot h^{(4+n)/(2+n)}$$

where $h$ is the radius of the circular surface.

The constant in these equations has been expressed by Pasquill (3) in terms of determinate physical quantities:

$$\Lambda \propto B \cdot \Lambda^{2n/(2+n)} \cdot K^{4(1-n)/(2+n)}$$
\[ B = \frac{M \rho D^{2n/(2+\alpha)}}{RT} \]  

(4)

where \( \Lambda \) is the kinematic viscosity of air (taken to be 0.147 cm\(^2\) sec\(^{-1}\)). \( \kappa \) is the von Karman constant (= 0.4); \( M \) is the molecular weight of the diffusing vapour; \( \rho \) is its vapour pressure; \( D \) is the vapour diffusivity (taken to be in the range 0.05-0.1 cm sec\(^{-1}\) for most molecules, except those which are strongly polar; \( R \) is the gas constant; \( T \) the absolute temperature.

For \( \nu \), Sutton's mean value, 0.25, may be taken without introducing an error greater than 10%.

With this value for \( \nu \), the exponent of \( D \) in (4) is 0.2 so that for all practical purposes (except in the case of water) it is reasonable to put \( D^{2n/(2+\alpha)} = 0.56 \).

Inserting the numerical values into equations (1) and (2) yields

\[ E(\omega, x, y) = 1.2 \times 10^{-2} \left( \frac{M \rho}{T} \right)^{0.15} \left( \frac{x^{0.99}}{y} \right) \]  

(5)

and

\[ E(\omega, x, y) = 3 \times 10^{-3} \left( \frac{M \rho}{T} \right)^{0.15} \left( \frac{x^{0.99}}{y} \right) \]  

(6)

with all quantities expressed in CGS units.

These expressions can be simplified by expressing the evaporation rate as volume of vapour produced in unit time at a chosen temperature (say 20°C) so that \( \frac{\rho}{T} \) vanishes.

When this is done the relationship can be expressed graphically by plotting the wind speed against the quantities

\[ F_1 = \frac{E(\omega, x, y)}{\rho x^{0.99}} \]  

(7)

and

\[ F_2 = \frac{E(\omega, x, y)}{\rho x^{0.99}} \]  

(8)

This has been done in Fig. 1.

Whence the rates of evaporation may be determined

\[ E(\omega, x, y) = F_1 \left( \frac{\rho x^{0.99}}{y} \right) \]  

(9)

\[ E(\omega, x, y) = F_2 \left( \frac{\rho x^{0.99}}{y} \right) \]  

(10)

With \( E \) expressed in litres per minute; \( \rho \) in mm. of mercury; and \( x, y \) and \( \lambda \) in metres.

The equations show that in the case of a rectangular surface the evaporation rate is greatest when the wind is blowing along the short axis.

A spillage of a volatile liquid thus provides a continuous steady source of vapour to be treated as a point or an
Vapour from a liquified gas

A liquified gas under pressure at ambient temperature is naturally at a temperature above its normal boiling point. Therefore when it is released there is an immediate flash-off of part of the liquid converted into vapour. The process is nearly instantaneous and therefore adiabatic, the heat of vapourisation being supplied by the sensible heat of the liquid.

As far as can be ascertained the question of the amount flashing-off has not been dealt with in the literature. The quantity can be obtained by equating the energy required to vapourise the liquid and that available from cooling the liquid.

Since the system is already at ambient temperature the only heat required for vapourisation is the latent heat. As the liquid cools below ambient temperature the vapour produced from it is at the temperature of the liquid. Subsequently the vapour is warmed by mixing with the air but this does not affect the vapourisation process.

When the residual liquid has cooled to its normal boiling point there is an abrupt fall in the rate of vapourisation and the flash-off phase ceases. The heat available is therefore that from cooling the residual liquid down to its boiling point.

Neither the latent heat of vapourisation nor the specific heat of the liquid are constant over the temperature range. The full data are not generally available. What may be found in reference books is the specific heat at or near the boiling point and the latent heat of vapourisation at the normal boiling point. If these are used and treated as constants no great error will be introduced compared with the uncertainties attached to subsequent calculations on the dispersion of the cloud.

Equating the energy changes gives

\[ l \, dW = s \, W \, dT \]  \hspace{1cm} (11)

where \( l \) is the latent heat; \( W \) the amount of liquid present; \( s \) the specific heat at \( T \) the temperature.

By integration, if \( l \) and \( s \) are constants

\[ W' = \exp \left[ \frac{s}{l} (T_{b} - T_{0}) \right] \] \hspace{1cm} (12)

where \( T_{b} \) is the normal boiling point and \( T_{0} \) the initial (ambient) temperature. It will be noted that \( (T_{b} - T_{0}) \) is a negative quantity. \( W' \) is the fractional amount of liquid remaining so that \( (1-W') \) is the fractional amount of liquid which vapourises.
This equation (12) will not be valid if the temperature is near the critical since in this region the values of $l$ and $s$ change rapidly. However, it is reasonably accurate for the majority of the common liquified gases. For these it will be found that $(-w')$ generally lies in the range 0.2 to 0.33.

In certain cases better data may be available for $l$ and $s$ and a more precise equation be required.

If the two parameters can be expressed as linear relationships

$$s = k_1 T + \alpha$$
$$l = k_2 T + b$$

$k_1, k_2, \alpha$, and $b$ being constants, these may be inserted in equation (11) which on integration yields

$$\int_0^t \frac{\partial w}{\partial t} \, dt = \int_0^t \frac{\partial w}{\partial t} \, dt$$

After the flash-off the residual liquid, now at its normal boiling point, is in contact with the ground, e.g. in a bund. Since it will now be colder than the ground it will receive heat until the surface of the ground is cooled. Further vapour will be produced in the first few minutes but the quantity will be small compared with that of the initial cloud and may be ignored.

Subsequently, after a few minutes, the cold liquid in the bund will behave in the same manner as a spilled cryogenic liquid which is dealt with below. As such it can be taken as a continuous steady source.

Thus on spillage of a liquified gas there are two phases: An initial flash-off producing a large cloud as an instantaneous source followed by a steady state continuous source. The former is so much larger than the latter that it completely dominates all consideration of hazard areas on dispersion downwind.

Vapour from a cryogenic liquid

A cryogenic liquid is a liquified gas stored at ambient pressure and a very low temperature. Since it is, in effect, at its normal boiling point there is no flash-off when it is released. There is, however, a very rapid boil-off of part of the liquid when it comes into contact with the relatively hot ground and thus yields a large cloud of vapour which may be treated as an instantaneous source.

The boiling rate falls off as the ground cools. In practice it is found that after one minute the rate has fallen to about one tenth of the initial value. It is convenient, therefore, to take an arbitrary time interval of one minute after discharge as the basis for estimating the size of the "instantaneous" cloud.
This problem has been considered by various authors. Burgess and Zabetakis (4) deduced a relationship which gives an approximate estimate of the initial rate

\[ V = \frac{\Theta (T_a - T_0)}{\rho l (\kappa \Phi t)^{1/3}} \]  

where \( V \) is the volume of liquid per unit area (or depth of liquid) which evaporates in one second; \( \Theta \) is the thermal conductivity and \( \Phi \) the thermal diffusivity of the ground; \( T_a \) the original temperature of the ground; \( T_0 \) the temperature of the liquid; and \( t \) the time in seconds. Values for \( \Theta \) and \( \Phi \) for various substrates are to be found in Carslaw and Jaeger (5).

Further progress was made by Walker, Wesson and Sliepecevich (6) and Parker (7) who produced equations for the continuing evaporation rate. These equations are complex and there is considerable doubt regarding their validity. A simplified argument leads to an equation which seems to give results of sufficient accuracy for practical purposes.

In an initial short period the quantity of heat extracted from the ground will be

\[ H_e = \kappa_s S_e \rho (T_a - T_0)^2/2 \]  

where \( H_e \) is the heat given up by the ground; \( \kappa_s \) a constant depending on the heat transfer characteristics of the ground; \( S_e \) the specific heat of the ground; \( \rho \) the density of the ground; \( T_a \) its initial temperature; and \( T_0 \) the temperature of the liquid.

This heat will vapourise a quantity of the liquid given by

\[ Q = \frac{H_e}{l} = \kappa_s S_e \rho (T_a - T_0)^2/2 \]  

where \( l \) is the latent heat of vapourisation, and \( Q \) will be the weight evaporated.

Values for \( S_e \) and \( \rho \) may be obtained from Carslaw and Jaeger (5). To obtain a numerical value for the constant the experimental results on liquified methane, hydrogen and nitrogen obtained by Burgess and Zabetakis (4) and Zabetakis and Burgess (8) and Parker's (7) calculations may be used.

Combining these values we obtain

\[ \frac{Q_1}{l} = \kappa_u (T_a - T_0)^2/ \]  

where \( Q_1 \) is the weight in grams produced in the first minute from each square centimetre of surface and \( \kappa_u \) is a constant having characteristic values for different substrates. Typical values are

<table>
<thead>
<tr>
<th>Substrate</th>
<th>( \kappa_u )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average soil</td>
<td>( 7.1 \times 10^{-4} )</td>
</tr>
<tr>
<td>Dry sandy soil</td>
<td>( 1.8 \times 10^{-4} )</td>
</tr>
<tr>
<td>Moist (8% water) sandy soil</td>
<td>( 4.3 \times 10^{-4} )</td>
</tr>
<tr>
<td>Concrete</td>
<td>( 7.5 \times 10^{-4} )</td>
</tr>
</tbody>
</table>
The cloud of vapour which is produced during this phase is so much larger than any subsequent generation of vapour that it is of primary significance in calculating the hazard area.

After the initial boil-off the rate falls ultimately to a quasi-steady state. In the case of a large spillage it may take an hour or more before the steady state is reached. There does not seem to be any simple method of calculating the rate during this intermediate period. The best that can be done is to interpolate on a logarithmic scale between the initial and steady state rates.

The steady state rate depends upon the latent heat of the liquid and the thermal characteristics of the substrate. An equation corresponding to equation (16) can be obtained

\[ Q = \kappa_s \Theta (T_g - T_0) / \ell \phi \]  

Inserting numerical values gives

\[ Q_s = \kappa_b (T_g - T_0) / \ell \]  

where \( Q_s \) is the weight in grams evaporating per minute from each cm\(^2\) of surface. Values of the constant \( \kappa_s \) are

<table>
<thead>
<tr>
<th>Substrate</th>
<th>( \kappa_s )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average soil</td>
<td>1.5 \times 10^{-2}</td>
</tr>
<tr>
<td>Dry sandy soil</td>
<td>6.3 \times 10^{-2}</td>
</tr>
<tr>
<td>Moist (8% water) sandy soil</td>
<td>1.1 \times 10^{-2}</td>
</tr>
<tr>
<td>Concrete</td>
<td>1.5 \times 10^{-2}</td>
</tr>
<tr>
<td>Sandstone</td>
<td>2.6 \times 10^{-2}</td>
</tr>
<tr>
<td>Limestone</td>
<td>2.1 \times 10^{-2}</td>
</tr>
<tr>
<td>Granite</td>
<td>2.4 \times 10^{-2}</td>
</tr>
<tr>
<td>Wood (spruce)</td>
<td>1.5 \times 10^{-3}</td>
</tr>
</tbody>
</table>

**DISPERSON OF VAPOUR**

It is necessary now to consider the dispersion of the cloud of vapour as it travels downwind. There are two separate situations: the large instantaneous cloud from a flash-off or boil-off; and the continuing source from a volatile liquid or the steady state after a flash-off or boil-off.

The vapour travels downwind becoming mixed with air by turbulence. In this process molecular diffusion is so slow that it may be ignored. In the classic treatments it is assumed that the vapour is of approximately the same density as air so that there are no gravitational effects. The problem has not been satisfactorily solved for the case when
the density difference is so great that gravitational effects cannot be ignored. Although much work has been done on buoyant plumes from smoke stacks, the problem of a discharge of heavy vapour is somewhat different.

The basic equations of dispersion are those developed in 1932 by Sutton (9), amplified by his experimental data in 1947, Sutton (10) and summarised in his book (2). Pasquill (11) has produced a modified treatment, but, since Sutton's method is most used, it is proposed here to confine discussion to his treatment. Gifford (12) gives useful alignment charts for the solution of specific problems. Other useful charts and discussions are given in publications by the U.S. Weather Bureau (13) and the U.S. Environmental Protection Agency (14). An edition, dated 1961, of Meteorology and Atomic Energy published by the U.S. Atomic Energy Commission (15) favours Pasquill's treatment.

Dispersion from an instantaneous source

For an instantaneous point source Sutton's basic equation is

\[ x(x_0, y_0, z) = \frac{2Q}{\pi^{3/2} C^2 \omega} \exp \left[ - \left( \frac{x_0^2 + y_0^2 + z^2}{C^2} \right) \right] \]  

(20)

\( x, y \) and \( z \) are the linear coordinates measured from an origin at the centre of the cloud as it moves downwind at the mean velocity \( \omega \) of the wind; \( x \) is the axis along the line of the wind; \( y \) the horizontal axis at right angles and \( z \) the vertical axis; \( x(x_0, y_0, z) \) is the concentration of vapour at the point \( x_0, y_0, z \) after time \( \omega \); \( Q \) is the quantity of vapour originally at the source; \( C \) is a diffusion coefficient which is taken to be the same in all three directions, that is isotropic conditions; and \( \omega \) is the meteorological turbulence factor. The numeral \( \omega \) appears in the numerator because the centre of the cloud is taken to have its origin at ground level and to remain at ground level as the cloud moves - that is to say the cloud is a hemisphere in which concentrations are twice what they would be in the case of a spherical cloud expanding equally in all directions.

The big advantage of Sutton's treatment is that \( C \) and \( \omega \) may be determined from meteorological parameters. Numerical values have been assigned and verified by experiment. The following values are generally accepted for diffusion over a smooth flat surface

<table>
<thead>
<tr>
<th>Meteorological condition</th>
<th>( C ) (metres)</th>
<th>( \omega ) (dimensionless)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lapse</td>
<td>0.2</td>
<td>0.17</td>
</tr>
<tr>
<td>Neutral</td>
<td>0.14</td>
<td>0.25</td>
</tr>
<tr>
<td>Inversion</td>
<td>0.09</td>
<td>0.35</td>
</tr>
</tbody>
</table>

The form of equation (20) indicates that it is the three dimensional Gaussian distribution. That is to say the concentration is distributed normally with a standard deviation which can be determined from the equation and with its maximum at the centre of the cloud, i.e. at \( x=y=z=0 \).
It is a simple matter to show that the standard deviation of the concentration, $\sigma_x$, is given by

$$\sigma_x = (\bar{\omega} t)^{(1-m)/2} \cdot C / \sqrt{2} \tag{21}$$

Inserting the values for $C$ and $\omega$ for neutral conditions gives

$$\sigma_x = 0.1 (\bar{\omega} t)^{0.97} \tag{22}$$

This function has been plotted in Fig. 2.

Maximum distance. The problem which arises most frequently is to determine the maximum distance to which an inflammable mixture can be carried by the wind. This will, of course, be the distance from its origin when the concentration at the centre of the cloud is equal to the lower flammable limit of the air/vapour mixture.

Centre of cloud. The position of the centre of the cloud from its origin at time $t$ is given by $(\bar{u}t)$

The concentration at the centre of the cloud at time $t$, $X(\omega, \omega, \omega)$, is given by putting $\sigma_x = 0$ in equation (20) with the relevant values of $C$ and $\omega$. By doing this the following equations are obtained for the three meteorological conditions

Lapse

$$X(\omega, \omega, \omega) = \frac{u\gamma Q}{(\bar{\omega}t)^{2.94}} \tag{23}$$

Neutral

$$X(\omega, \omega, \omega) = \frac{u\mu Q}{(\bar{\omega}t)^{2.42}}$$

Inversion

$$X(\omega, \omega, \omega) = \frac{u\gamma Q}{(\bar{\omega}t)^{2.94}}$$

where $Q$ is the total quantity of vapour in the cloud, in grams; $\bar{\omega}$ is mean wind speed in m sec$^{-1}$; $t$ is the time in seconds; $X$ is the concentration in grams m$^{-3}$. These have been plotted in Fig. 3.

By putting $X(\omega, \omega, \omega)$ equal to the lower flammability concentration the appropriate equation can be solved simply to give the distance $(\bar{\omega}t)$.

It will be noted that this distance is independent of the wind speed. At different wind speeds the time of arrival will be different so that $(\bar{\omega}t)$ remains constant.

Dimensions of cloud. Given the position of the centre of the cloud $(\bar{\omega}t)$, the standard deviation (by equation 22) and either the concentration at the centre (by equation 23) or the total quantity of vapour $Q$, it is possible to calculate all the relevant properties of the cloud by the techniques ordinarily applied to normal distributions.

For example it may be desirable to determine the radius of the cloud at some position $(\bar{\omega}t)$. Since the normal distribution extends to infinity it is necessary to define the position of the perimeter of the cloud - say where the concentration $X$ is equal to some chosen value, e.g. the lowest objectionable value.
Since the cloud is symmetrical we may choose any axis putting any two of $x$, $y$, or $z$ equal to zero, to obtain $X(x,0,0)$.

The concentration at the centre, $G$, is calculated by equation (23), then from the properties of a normal distribution

$$X(x,0,0) = G \exp \left[ -\frac{x^2}{2\sigma_x^2} \right]$$

where $\sigma_x$ is given by equation (22).

By solving this equation $X$, the radius, may be obtained for any chosen value of $X$.

Integrated dosage. In the case of the release of a toxic vapour it is desirable to know the total integrated dosage ($\int D$) to which a person might be exposed. This is a maximum along the axis of the cloud's path.

This may be obtained by integrating equation (20) along the $x$ axis, yielding

$$\int D = 2Q/\pi C^2 \exp \left( -C^2 \right)$$

Position of origin of cloud. The above equations are for a point source. In actuality the source is not a point. The released cloud is of some size. So that its origin as a virtual point would be some distance upwind and the distances given by the equations would tend to be over-estimated. Holland (16) discusses this point and suggests a method of estimating the position of the virtual point source. However, in actuality, one knows very little about the size of the cloud at the moment of release. At a minimum it would be that of a hemisphere of pure vapour but mixing with air must occur to an unknown extent. Since the error resulting from treating the source as a point is on the side of safety it is prudent to regard it as such and not to introduce a spurious impression of accuracy.

Instantaneous concentrations. In these, and subsequent, equations the concentration $X$ is the time mean concentration. In a real cloud there is a microstructure as can be seen by looking at the smoke plume from a stack. At any point there are momentary fluctuations. It has been estimated that the maximum instantaneous concentration is about 1.7 times the time mean concentration. When there are local eddies in a cloud there may be small isolated zones of a concentration above the lower flammable limit although the mean concentration is below the limit. Ignition of such a zone may propagate to adjacent zones and thence back to the main mass of the cloud. It is necessary to take this into account in assessing hazard areas.

Dispersion from a continuous source

The evaporation of a volatile liquid and the steady-state after an initial flash-off or boil-off are continuous sources. For
such some modifications are required to equation (20).

In the first place, for obvious reasons, there is no "fore and aft" diffusion in a continuous emission. Hence \( x \) disappears from the equation. At any point the amount of vapour is the integral in respect of \( x \).

Whereas in the case of an instantaneous cloud it was convenient to take the centre of the cloud as origin, in the case of a continuous source there is no centre in that sense so the distance along the direction of the wind is measured from the source. This is designated \( x_1 \) with a subscript \( l \) to distinguish it and it replaces \( \bar{u} \).

Since the cloud is continuous the time element, \( t \), also disappears. It is necessary also to replace \( Q \) by \( R \), the quantity of vapour emitted in unit time.

**Continuous point source**

Making these changes and integrating the equation for isotropic conditions becomes

\[
X(x_1, y_1, z_1) = \frac{2R}{\pi c^2 \bar{u} x_1^{2-\alpha}} \exp \left[ -x_1^{(2-\alpha) / \alpha} \left( \frac{y_1^2 + z_1^2}{c^2} \right) \right]
\]

where \( R \) is the quantity of vapour emitted in unit time. The maximum concentration will be along the axis with \( y_1^2 + z_1^2 = 0 \) giving

\[
X(x_1, 0, 0) = \frac{2R}{\pi c^2 \bar{u} x_1^{2-\alpha}}
\]

Inserting the values of \( C \) and \( \bar{u} \) for neutral conditions gives

\[
X(x_1, 0, 0) = \frac{32R}{(\bar{u} x_1^{1/3})}
\]

This function has been plotted in Fig. 4, for selected value of \( \bar{u} \) the mean wind speed.

It will be noticed that the concentration is inversely proportional to the wind speed. It will be recalled that equations (5) and (6) showed that the rate of evaporation from a pool of volatile liquid is proportional to \( \bar{u}^{5/2} \) so that at any point downwind the concentration of vapour is proportional to \( \bar{u}^{5/2} / \bar{u} = \bar{u}^{-3/2} \). This means that for a spillage of a volatile liquid the vapour concentration decreases as the fifth root of the wind speed, that is to say a ten-fold increase in wind speed decreases the concentration by about one third.

In contrast the rate of evaporation of a cryogenic liquid is independent of the wind speed (by equation 19). Hence the concentration of vapour is inversely proportional to the wind speed.

The standard deviation of concentration in the \( y \) and \( z \) directions is the same as before, namely
whence the characteristics of the cloud may be calculated.

Continuous line source of infinite length

A linear cross-wind source of great length may be treated as an infinitely long source with no sideways diffusion, i.e. \( y \) disappears. The relevant equation is then

\[
\chi_{x}(x, z) = \frac{2R'}{\pi \sigma x_{0}^{2}} e^{\gamma^{2} \left( -x_{0}^{2} - z^{2} / \sigma^{2} \right)}
\]

The maximum concentration is at ground level, \( z = 0 \) and is uniform across the whole width of the cloud

\[
\chi_{x}(x, 0) = \frac{2R'}{\pi \sigma x_{0}^{2}}
\]

which for neutral conditions gives

\[
\chi_{x}(x, 0) = \frac{8R'}{(\bar{u} x_{0}^{0.5})}
\]

with the standard deviation in the \( z \) direction the same as before.

Continuous line source of finite length

When the cross-wind linear source is of finite length the equations become more complicated because there is sideways diffusion towards the edges. Sutton (17) gives an explicit equation for this situation.

The maximum concentration is at ground level along the axis, i.e. when \( y = z = 0 \). Inserting these values in Sutton's equation the maximum concentration at the point \( x_{0} \) is given for neutral conditions by

\[
\chi_{x}(x_{0}, 0, 0) = \frac{8R'}{(\bar{u} x_{0}^{0.5})} \left[ \text{erf} \left( \frac{L}{0.28 x_{0}^{0.5}} \right) \right]
\]

where \( L \) is the length of the cross-wind source.

The error function, in square brackets, may be regarded as the correction factor by which the term outside the bracket (which is the concentration from an infinitely long line source) must be multiplied to account for the losses by sideways diffusion.

Values of this "diffusion factor" have been plotted in Fig. 5 for several values of \( L \).
SYMBOLS USED

A = a complex function defined by equations (3) and (4)
α = a constant
B = a complex function defined by equation (4)
b = a constant
C = Sutton's diffusion coefficient
D = vapour diffusivity
E = evaporation rate; \( E_{xy} \) = evap. rate at point \( x,y \)
\( E_{(x,y)} \) = evap. rate from rectangle \( x,y \); \( E_c \) = evap. rate from circle of radius \( c \)
\( E_1, E_2 \) = functions defined by equations (7) and (8)
G = concentration of vapour at centre of cloud
H = heat given up by the ground in initial period
K = Von Karman constant
\( k, k_0 \) = constants
L = cross-wind length of a linear continuous source
L = latent heat of vapourisation
M = molecular weight
n = meteorological turbulence factor
p = vapour pressure
Q = quantity of liquid evaporating
\( Q_1 \) = weight in grams of liquid evaporating in first minute from 1 cm²
\( Q_t \) = weight of liquid evaporating per minute per cm² during steady state
\( Q' \) = quantity of vapour emitted in unit time
r = radius of a circular surface
\( \lambda = \) specific heat of the liquid
\( \lambda_e = \) specific heat of the ground
T = temperature; subscripts \( 0 \) = initial; \( g \) = ground; \( \nu \) = b.p. of liquid
t = time
\( \bar{u} \) = mean wind velocity
V = volume of liquid evaporating from unit area in one second
W = initial weight of liquid
\( W' \) = fractional weight of liquid remaining after flash-off
X = distance from centre of hemispherical cloud along line of wind direction
\( x_1 \) = distance from source along line of wind direction
\[ u = \text{distance along cross-wind horizontal axis} \]
\[ z = \text{distance along vertical axis} \]
\[ \tau = \text{total integrated dosage} \]
\[ \Theta = \text{thermal conductivity of the ground} \]
\[ \lambda = \text{kinematic viscosity of air} \]
\[ \rho = \text{density of the ground} \]
\[ \sigma = \text{standard deviation of concentration of vapour} \]
\[ \varphi = \text{thermal diffusivity of the ground} \]
\[ X = \text{concentration of vapour} \]
\[ X(\omega, \beta, \tau) = \text{concentration of vapour at point } \omega, \beta, \tau \text{ and time } \tau \]

REFERENCES

FIG. 1. EVAPORATION OF VOLATILE LIQUID SPILLAGE
FIG. 2. STANDARD DEVIATION OF CONCENTRATION
FIG. 3. CLOUD FROM FLASH-OFF
MAXIMUM CONCENTRATION AT CENTRE
RELATIVE TO DISTANCE

\[ q \bar{t} - \text{QUANTITY OF VAPOUR/CONCENTRATION (m}^3\text{)} \]

\[ \bar{u}t - \text{DISTANCE (km)} \]

- \( q \) = Quantity flashed-off (grams)
- \( x \) = Concentration (g/m\(^3\))
- \( \bar{u} \) = Mean wind speed (m/sec)
- \( t \) = Time (sec)
$\bar{u}$ = Mean wind speed (m/sec)
$R$ = Source strength (g/sec)
$X$ = Concentration (g/m$^3$)

Fig. 4. Continuous Point Source
Maximum Concentration v Distance
(Neutral Atmosphere)
FIG. 5. DIFFUSION FACTOR
CONTINUOUS FINITE LINE SOURCE

$L = \text{Length of source (m)}$

$\text{erf}(L/0.28 x^{0.5}) = \text{DIFFUSION FACTOR (dimensionless)}$

$x = \text{DISTANCE FROM SOURCE (m)}$

$L = 100$
$L = 40$
$L = 20$
$L = 10$